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DYNAMIC ANALYSIS OF THE TETHERING CABLE PORTION OF A HIGH ALTITUDE TETHERED BALLOON SYSTEM UNDER FULLY DEPLOYED NDITIONS

FINAL REPORT

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Prepared by

NESCO Staff

Project Scientist - A. Soldate

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FOREWORD

The theoretical development of the determination of the steady-state cable configurations, the treatment of the vortex shedding loading, and the work presented in Appendix B are due entirely to Dr. S. Fersht. Programs were developed by both Mr. H. L. Butler and Mrs. Mariann Moore. The program manager was Dr. A. M. Soldate. Valuable guidance was given to the program by Dr. C. Dudley Fitz.

This program was accomplished under the sponsorship of the Advanced Research Projects Agency (ARPA) and was a portion of ARPA's effort to investigate several problems related to the design characteristics and operational features of a high altitude tethered balloon system.

SYNOPSIS

A computer study has been made of nonsteady aerodynamic loadings on a long, cylindrical cable of the continuous glass fiber-resin type used as a tether for a balloon at altitudes of approximately 100,000 feet. No important interactions between torsional, longitudinal, and lateral modes were found. Furthermore, the effects of lateral loadings from gusts or vortex sheddings were found to be unimportant. Computer programs are presented that enable computations to be made of cable motions resulting from localized gust loadings and from vortex shedding phenomena.

Certain laboratory and field tests are recommended for further studies of the effectiveness of the continuous glass fiber-resin cable as a balloon tether.

CONTENTS

1. INTRODUCTION

Preliminary system analyses have indicated the technical possibility of establishing and maintaining tethered balloon systems at altitudes in the order of 100,000 feet. The practical feasibility of such systems is critically dependent, however, upon the characteristics and properties of the tether cable. Cable width must be kept small to minimize the aerodynamic drag. At the same time, the cable cross section must be sufficient to carry the weight of the portion of the cable below or to withstand the accumulated drag on the cable above. A simple calculation shows that constant diameter cables constructed of ordinary steel are not capable of supporting their own weight over the height of interest. Thus, strength, weight, and drag properties are intermeshed, and trade-offs between these properties must be made.

If cable weight were the only problem, a possible solution could be obtained by tapering or stepping down the cable diameter at lower altitudes to decrease the weight to be supported. Drag force must also be considered however, and these forces can only be balanced by a restraining horizontal force at the ground terminal end of the cable. Since the drag forces are cumulative (increasing toward the bottom), a tapered cable must increase its diameter at the lower end. The resulting hourglass shape would be correct for only one wind profile. The hourglass shape is also difficult and expensive to fabricate and to operate.

Similar results are encountered in attempting to use balloons or kite devices at intermediate altitudes to help support the cable weight. Although such devices reduce the load requirements on the upper cable, the drag forces are increased considerably, requiring an increase in cable size. Furthermore, the varying lift provided by aerodynamic devices will create rapidly changing geometric conditions, possible instabilities, and certainly will increase the complexity of the winch-cable subsystem.

Another approach for solving the cable problem lies in the improvement of the cable materials. In searching for a suitable material, a high strength/weight characteristic is one of the first properties to consider. Fiberglass with a strength to weight ratio many times that of steel is a strong candidate for the high altitude balloon cable. Owens Corning has developed a combination of glasses which, drawn in individual filaments, displays a strength in the order of 10^6 psi. The Owens Corning scientists have also demonstrated their ability to produce long multi-strand fiberglass cables. Under ARPA/NOTS funding in the fall of 1964, Owens Corning drew a cable consisting of 30,000 individual fibres with an overall diameter in the order of 0.1 inch and a total length in excess of 80,000 feet. The group of fibers, bonded together with epoxy, has a strength over the total area including the voids in the order of 0.25×10^6 psi. Strength to weight ratio of this glass cable exceeds that of ordinary steel cables by a factor of twenty. Furthermore, cost is estimated as only 10 cents per foot. This constant diameter cable should easily carry the required weight of the cable at the top end and resist the static drag load at the ground terminal point.

Fiberglass has, however, a serious disadvantage in its weakness to compressive and shear loadings. This weakness was clearly demonstrated in a test flight accomplished in December 1964. In this test a sudden failure of the cable occurred while the cable was paying out with a rapidly rising balloon. Location and cause of the initial failure were not readily apparent. It was interesting to note, however, that the sudden release of tension apparently allowed a compressive wave to be generated which, as it propagated along the cable, caused the cable to puff out the individual filaments at many locations. The cable broke into many sections and fell as shards.

This experience illustrates the basic need for an analysis of the dynamic characteristics of the cable and determination of the possible occurrence and magnitude of compressive conditions in a balloon tethering cable. From this analysis, it will be possible to gain a better understanding of the required cable material properties and dynamic characteristics.

A complete analysis of the cable conditions can be accomplished by the following steps.

a) An analysis of cable statics and dynamic characteristics at the fully deployed condition (balloon at its float position) when the balloon and cable are subjected to an arbitrary wind profile.

b) An analysis of the balloon, cable, and cable control during launch and ascent for various wind conditions.

c) An analysis of the cable and cable control during recovery operations.

The study accomplished under the current program addressed the first of these steps.

As a result of this study, it was determined that, at a fully deployed state, the cable can be expected to be comparatively stable and that the naturally induced vibrations are not expected to seriously affect a cable constructed of fiberglass.

Field tests also have been suggested which will enable substantiation of the conclusions drawn from the theoretical investigations.

2. METHOD OF ATTACK

The general attack used on the problem of cable dynamics is to consider perturbed motions from the equilibrium (steady state) shape of the tether cable, the equilibrium state being determined by the balloon lift, balloon drag, balloon altitude and by the drag along the cable as determined by the wind velocity profile.

The dynamic equations employed are similar to those used by NESCO for an extensive and detailed numerical analysis of the riser and drill string system of Project Mohole (Ref. 1). Accordingly, NESCO's previous experience in appropriate numerical analysis techniques is directly applicable.

The determination of the steady state profile will be discussed first.

3. STEADY-STATE CABLE PROFILE

Practical considerations dictated the treatment of cable dynamics in two-dimensional space coordinates rather than development of a three-dimensional steady-state program. (The treatment of the steady-state profile can, however, be easily generalized to three dimensions.)

Geometrical parameters and coordinates are defined by Fig. 1. It may be noted (in this figure) that altitude is taken as the x-coordinate, and the lateral displacement is indicated by the y-coordinate; ds represents an element of the original length, and dS represents an element of the final length of the system such that

$$m_o ds = m dS \tag{1}$$

where

m_o is initial mass per unit length (kg/m)

and

$m = \frac{m_o}{C}$ is mass per unit length (in stretched condition)

with

$C = 1 + \frac{T}{EA}$ (cable stretch factor)

T = tension at element

E = Young's modulus

A = cross sectional area of cable

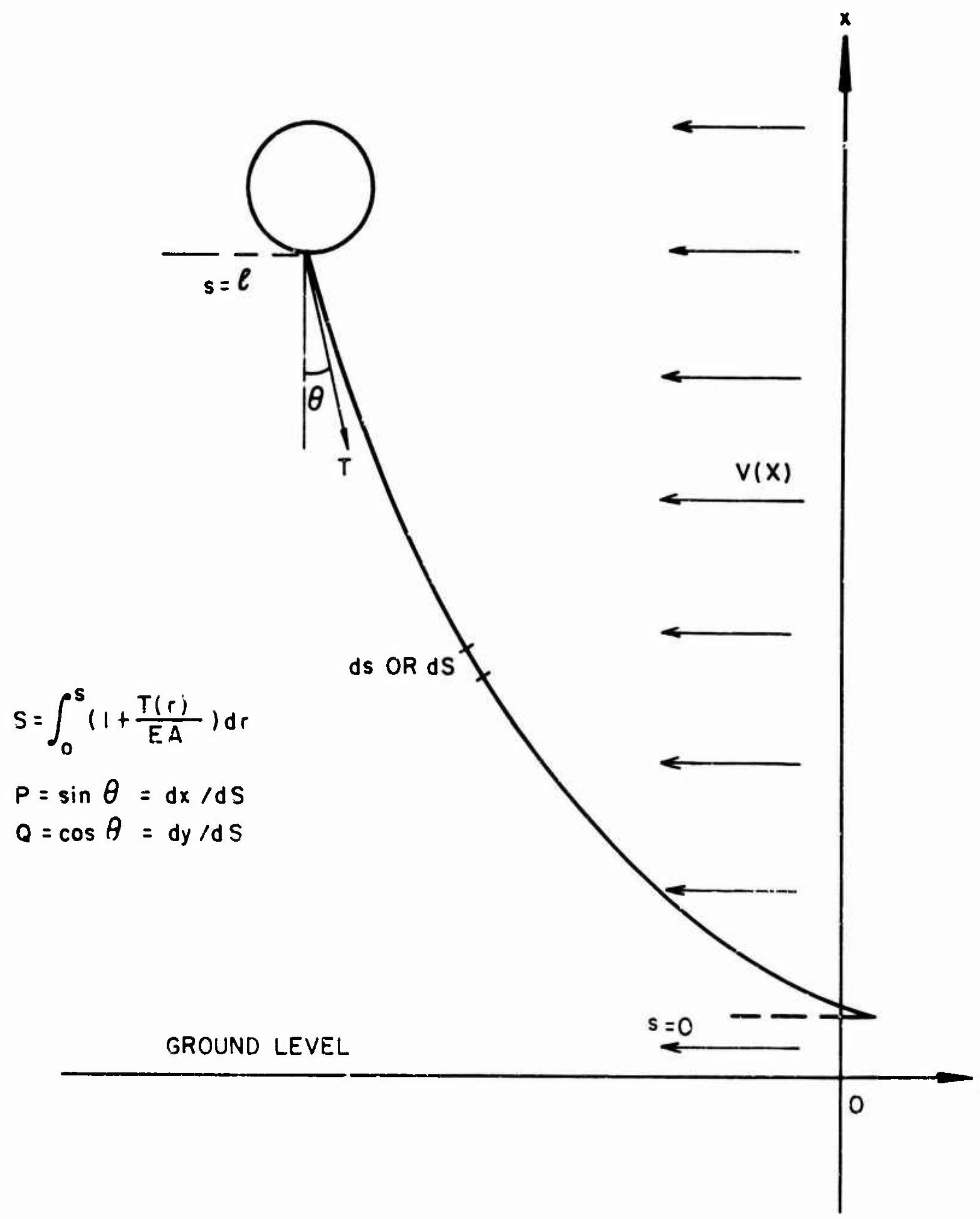

Figure 1
Definition of Axes

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Furthermore, the angle between the element axis and the vertical is designated as θ where

$$P = \sin\theta = \frac{dx}{dS} \tag{2a}$$

$$Q = \cos\theta = \frac{dy}{dS} \tag{2b}$$

Assuming a horizontal wind in the (x, y)-plane, the wind loading normal to the element axis is designated as p_n and

$$p_x = p_n Q \tag{3a}$$

$$p_y = -p_n P \tag{3b}$$

The equations of equilibrium are the following.

$$\frac{dx}{ds} = P \cdot C \tag{4a}$$

$$\frac{dy}{ds} = Q \cdot C \tag{4b}$$

$$\frac{dP}{ds} = \frac{Q \cdot C}{T} \cdot \left[\left(p_x + mg \right) Q - p_y P \right] \tag{4c}$$

$$\frac{dQ}{ds} = \frac{P \cdot C}{T} \cdot \left[p_y P - p_x + mg \quad Q \right] \tag{4d}$$

$$\frac{dT}{ds} = C \left[\left(p_x + mg \right) P + p_y Q \right] \tag{4e}$$

Assuming a horizontal wind of velocity v in the (x, y)-plane, the normal wind loading can be expressed as

$$p_n = \frac{1}{2}\frac{\rho(x)}{g} C_D(R)\, P^2 D v\, |v| \qquad (5)$$

where the Reynolds Number for the cable is taken as

$$R = \left|\frac{vD}{\nu} P\right| \qquad (6)$$

$$p_x = p_n Q \qquad (7a)$$

$$p_y = p_n P \qquad (7b)$$

The following are input parameters:

D	=	diameter of cable, meters
EA	=	force, kgf - kilogram weight
m_o	=	initial mass/unit length of cable, kg/m
g	=	acceleration of gravity at sea level, 9.81 m/sec^2 - used as a conversion factor to express all forces in kgf
ρ		mass density of air kg/m^3
ν		kinematic viscosity of air, m^2/sec

Boundary conditions are obtained from the following considerations:

a) For estimation purposes the balloon may be considered an a sphere filled with helium. Assuming equal pressure inside and outside the balloon, the static lift is

$$L = \pi/6\, D_b^3\, \rho(x_\ell)\, 25/29 \cdot g_x/g \qquad (8)$$

(g_x is the value of the gravitational constant at [illegible]

b) If the dead load, which includes the balloon's membrane and instrumentation, is W, the net static lift is

$$V_\ell = L - W \tag{9}$$

where W is an input parameter in Kgf; D_b is an input parameter (diameter of balloon) in meters and ℓ is the initial length of the cable in meters.

c) The wind load on the balloon is

$$H_\ell = \pi/8 \frac{\rho(x_\ell)}{g} D_b^2 \overline{C_D} (R_\ell) v(x_\ell) \cdot \left| v(x_\ell) \right| \tag{10}$$

in which the Reynolds Number for the balloon is designated

$$R_\ell = \left| \frac{v(x_\ell)}{\nu(x_\ell)} D_b \right| \tag{11}$$

where $\nu(x)$ is the kinematic viscosity as a function of altitude.

d) Boundary conditions are:

1) at $s = \ell$: An initial guess for x and y is made, then

$$T_\ell = \sqrt{H_\ell^2 + V_\ell^2}, \quad P_\ell = \frac{V_\ell}{T_\ell}, \quad Q_\ell = \frac{H_\ell}{T_\ell} \tag{12}$$

2) at $s = 0$: $x = 0$, $y = Y$

The following data are required as input to the main steady-state program.

a) A table of wind velocity as a function of altitude; the tabular velocities being denoted $\tilde{v}(x)$. The profile can be of arbitrary shape. The particular profile employed in the present study is shown in Fig. 2. The interpolated values of wind velocity are denoted $v(x)$.

b) Subprograms to compute the quantities $\rho(x)$--atmospheric density, $\nu(x)$--kinematic viscosity of atmosphere, $C_D(R)$--drag coefficient of cable (R is the Reynolds number), $\overline{C_D}(R_\ell)$--drag coefficient of balloon. Atmospheric properties are essentially those of the ARDC Standard Atmosphere (Ref. 2), the drag coefficients for a cylinder and a sphere are derived from the literature (Ref. 3). The expressions for these quantities used in the subprograms are:

1) Density $\rho(x)$ in kg/m^3

$$\rho(x) = 10^{(1/4) - (x/16000)}$$

2) Kinematic viscosity $\nu(x)$ in m^{2}sec^{-1}

$\nu(x)$ is defined over three ranges of x

$$x \leq 10000 \text{ meters} \qquad \nu = 10^{-4.823 + (x/26800)}$$

$$x \leq 17000 \text{ meters} \qquad \nu = 10^{-5.035 + (x/17100)}$$

$$x > 17000 \text{ meters} \qquad \nu = 10^{-5.2421 + (x/14250)}$$

3) Reynolds Number $R(x)$ for the cable

$$R(x) = \left| \frac{V_o(x)\, D\, P(s)}{\nu(x)} \right|$$

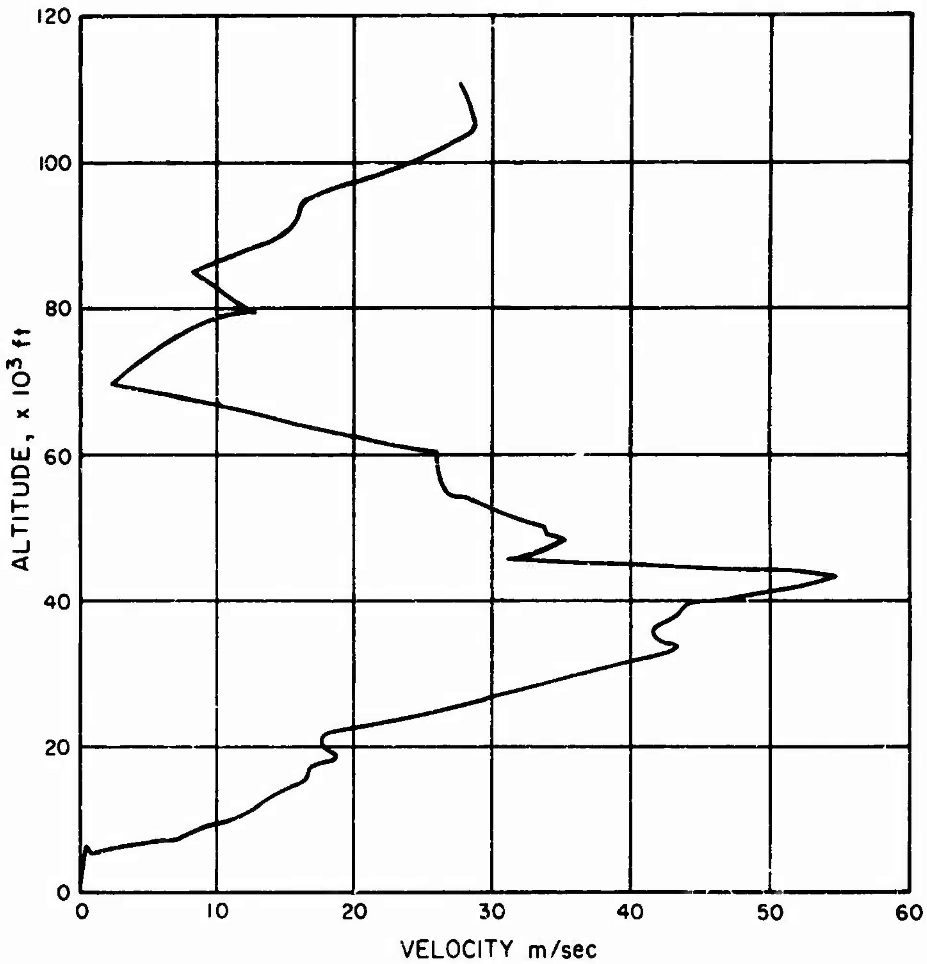

Figure 2

Two-dimensional wind profile used for steady-state profile determination

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4) Drag coefficient of cable $C_D(R)$

$C_D(R)$ is defined over five ranges of Reynolds number R.

$R \leq 2.23$	$C_D = 10.8\ R^{-.742}$
$R \leq 8.0$	$C_D = 9.15\ R^{-.526}$
$R \leq 1000.0$	$C_D = 4.95\ R^{-.232}$
$R \leq 10000.0$	$C_D = 1.0$
$R > 10000.0$	$C_D = 1.5$

Figure 3 shows the range of drag coefficients for the cable corresponding to the wind profile of Fig. 2. For a cable diameter of approximately 0.01 feet, the Reynolds Numbers are below (by two orders of magnatude) those for the transition region $(R < 3 \times 10^5)$. On the other hand, the diameters of balloons are of such a magnitude that transition will be encountered at certain altitudes. For a 5-foot diameter sphere, the transition region is encountered at about 50,000 feet.

5) Drag coefficient of balloon $\overline{C_D}(R_\ell)$

$\overline{C_D}(R_\ell)$ is defined over six ranges of Reynolds Number R_ℓ.

$R_\ell \leq 1.0$	$\overline{C_D} = 27.4\ R^{-.961}$
$R_\ell \leq 10.0$	$\overline{C_D} = 27.4\ R^{-.804}$
$R_\ell \leq 100.0$	$\overline{C_D} = 16.0\ R^{-.572}$
$R_\ell \leq 1130.0$	$\overline{C_D} = 7.25\ R^{-.4}$
$R_\ell \leq 10000.0$	$\overline{C_D} = 0.4$
$R_\ell > 10000.0$	$\overline{C_D} = 0.44$

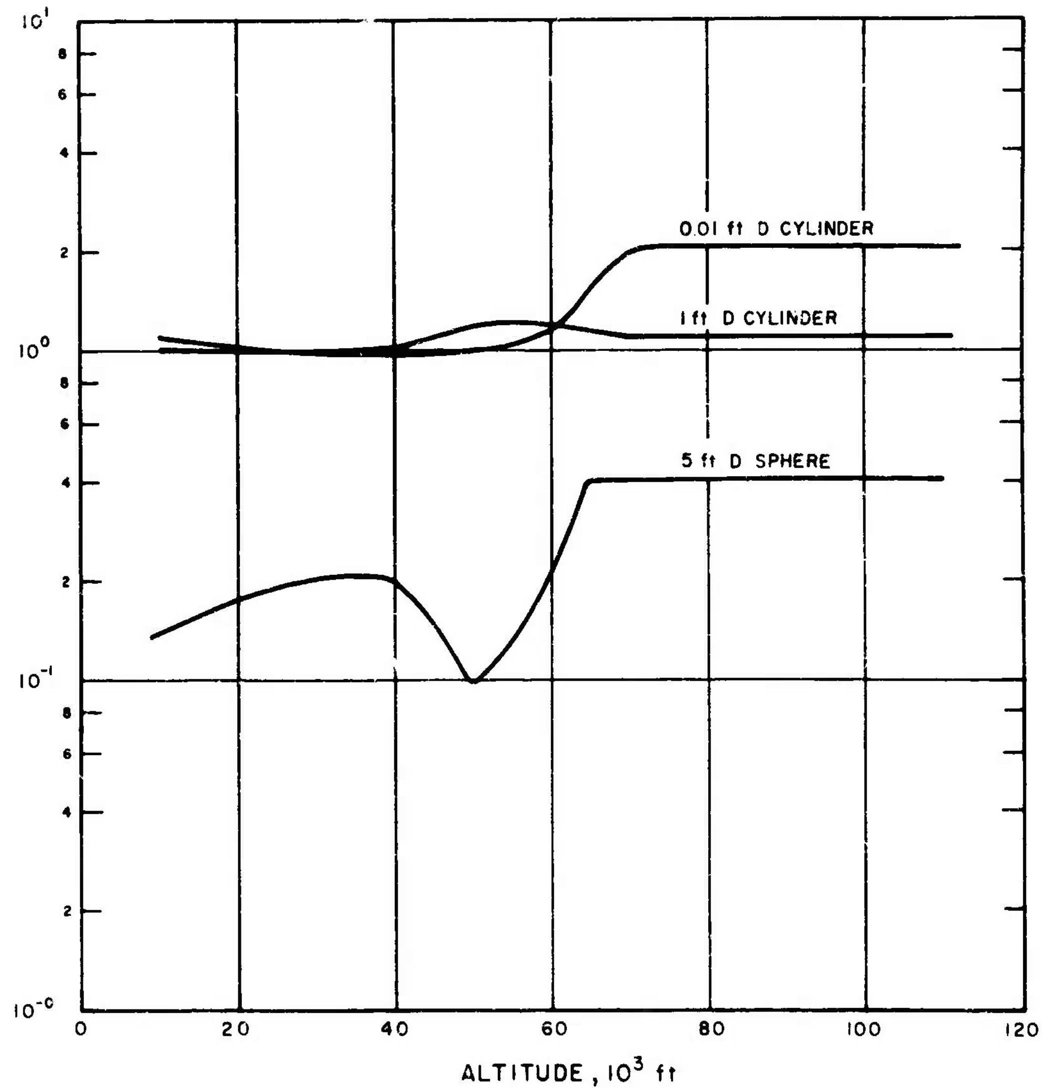

Figure 3
Drag coefficient as a function of altitude for cylindrical cables and small spheres corresponding to the wind profile of Fig. 2

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The method of solution is based on an iterative procedure employing a fourth order Runge-Kutta technique to integrate the system Eqs. 4a through 4e along the length of the cable. Assuming initial values for the altitude x and the lateral displacement y at $s = \ell$, the tension T and the dependent variables P and Q may be determined, thereby, giving a complete set of initial conditions at the upper end of the cable. Values for the altitude at the lower end of cable are obtained for various values of x at the upper end by the numerical integration scheme. The objective of this procedure is to choose the values of x at the upper end in such a manner as to force the sequence of values of x at the lower end x_o to converge to zero (ground level).

The values x_ℓ^1 and y_ℓ^1 are input parameters and the result of the first integration is x_o^1 (as well as y_o, T_o, P_o, and Q_o). Succeeding values of x_ℓ^i are determined by

$$x_\ell^{i+1} = x_\ell^i - \left|\frac{x_o^i}{2}\right| \tag{13}$$

for $i = 1, 2, \ldots$.

The iterative procedure is carried out until the condition

$$\left|x_o^i + 1\right| \leq \epsilon \tag{14}$$

is met, where ϵ is an input tolerance.

The equations of equilibrium given in the final coordinate system are

$$\frac{dx}{dS} = P \tag{15a}$$

$$\frac{dy}{dS} = Q \tag{15b}$$

$$\frac{dP}{dS} = \frac{Q}{T}\left[\left(p_x + mg\right)Q - p_y P\right] \tag{15c}$$

$$\frac{dQ}{dS} = \frac{P}{T}\left[p_y P - \left(p_x + mg\right)Q\right] \tag{15d}$$

$$\frac{dT}{dS} = \left(p_x + mg\right)P + p_y Q \tag{15e}$$

where m is taken as m_o and

$$S_\ell = \int_0^\ell \left(1 + \frac{T(s)}{EA}\right) ds \tag{16}$$

x, y, T, P, Q at $S = S_\ell$ are taken from the solution of the above iterative procedure.

With the use of the wind velocity profile shown in Fig. 2, a determination of the steady-state cable profile was made. The cable properties were the following.

Diameter	0.0025 meters
EA (Product of Youngs Modulus x cross sectional area)	3.451×10^4 kgf

E	10^7 psi*
m_o	8.0×10^{-4} Kg/m
Unstretched length of cable	36,000 meters

The balloon properties were taken as

Diameter	40 meters
Dead load	50 Kgf

(The properties of the balloon are not regarded as essential to the problem, but must be such that the system is stable.)

The results of the numerical example are the following: After five iterations the program was terminated giving the results

S	x	y	T	P	Q
m	m	m	kgf		
3.6×10^4	2.75×10^4	2.0×10^4	957.1	0.973	-0.230
0.0	13.5	4.12×10^4	746.6	0.427	-0.904

$S_f = 3.687 \times 10^4$

The final configuration was then determined and is plotted in Fig. 4. Agreement with the boundary conditions for x and y at the ground level was within 80 meters.

A flow chart, listing and other details of the program, is given in Appendix A. The output of this program is used as input to the dynamics program.

*Private conversation with Mr. Sheldon D. Elliot, Jr., gives a value of 7×10^6 psi per E for the glass fiber-resin cable. (Corresponds to a specific gravity of 1.6.)

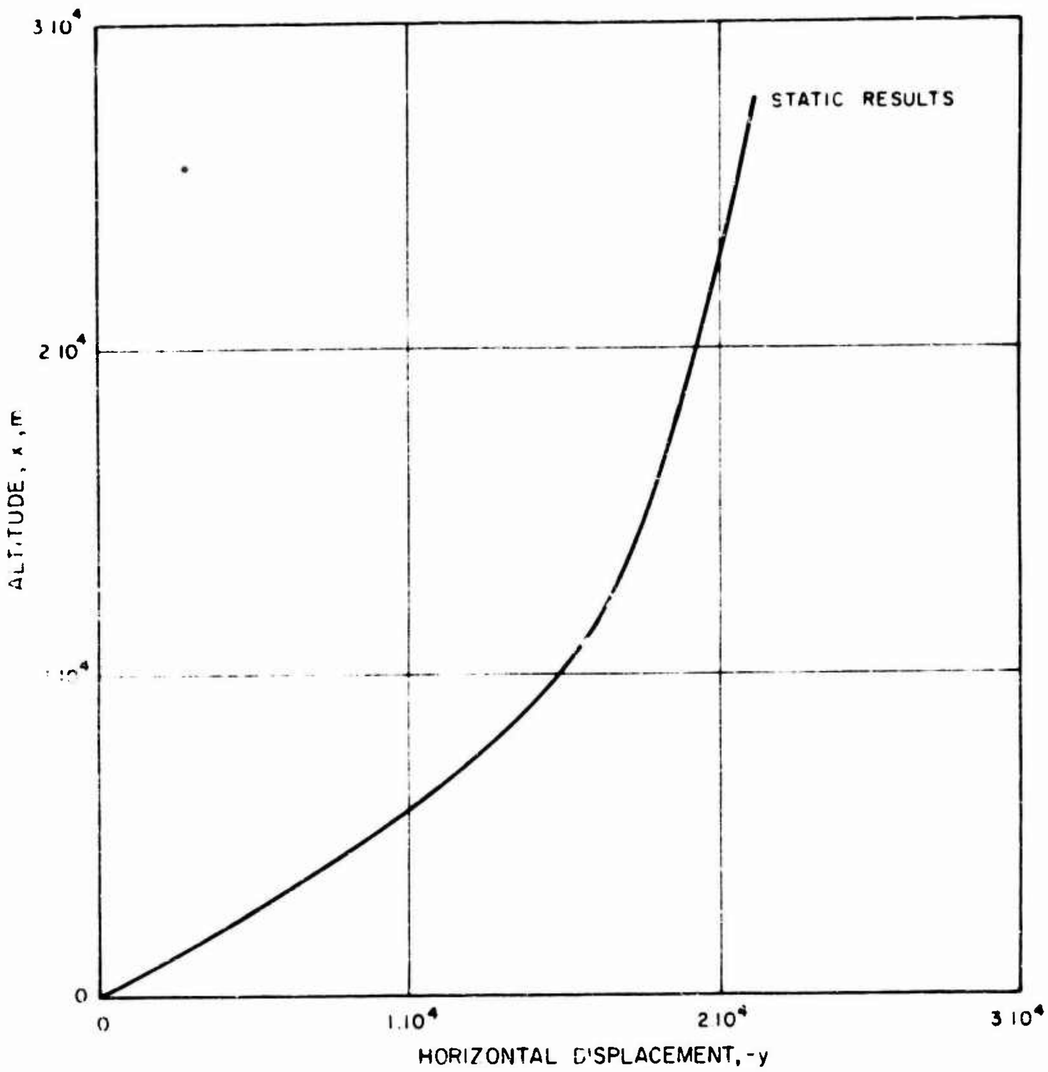

Figure 4
Static configuration of cable with representative air velocity profile

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4. CABLE DYNAMICS

The primary purpose of the study of cable dynamics is to consider possible situations that could lead to the existence of momentary stresses far in excess of those produced in the static or steady state loading condition. One possible situation is the presence of clear air turbulence in a certain altitude zone. Another situation is excitation by vortex shedding. A general question concerns possible interactions between various modes of cable vibration leading to stress amplifications. The possibility of the existence of important vibrational interactions will be discussed first.

With the coordinate system of Fig. 1 generalized to three dimensions by a z coordinate pointing up from the plane of the figure, and the assumption that cable is reasonably vertical, it can be shown that the coupling between torsional and lateral vibration modes is expressed by the following equations (Appendix B).

$$2G\left[\frac{\partial}{\partial x}\left(I\frac{\partial\theta}{\partial x}\right) + \frac{\partial y}{\partial x}\frac{\partial}{\partial x}\left(I\frac{\partial\theta}{\partial x}\frac{\partial y}{\partial x}\right) + \frac{\partial z}{\partial x}\frac{\partial}{\partial x}\left(I\frac{\partial\theta}{\partial x}\frac{\partial z}{\partial x}\right)\right] + m_x = J\frac{\partial^2\theta}{\partial t^2} \tag{17}$$

$$\frac{2\partial^2}{\partial x^2}\left(GI\frac{\partial\theta}{\partial x}\frac{\partial z}{\partial x}\right) - \frac{\partial}{\partial x}\left(N\frac{\partial y}{\partial x}\right) - p_y = -m\frac{\partial^2 y}{\partial t^2} + \frac{\partial}{\partial x}\left(J\frac{\partial z}{\partial x}\frac{\partial^2\theta}{\partial t^2}\right) \tag{18}$$

$$-\frac{2\partial^2}{\partial x^2}\left(GI\frac{\partial\theta}{\partial x}\frac{\partial y}{\partial x}\right) - \frac{\partial}{\partial x}\left(N\frac{\partial z}{\partial x}\right) - p_z = -m\frac{\partial^2 z}{\partial t^2} - \frac{\partial}{\partial x}\left(J\frac{\partial y}{\partial x}\frac{\partial^2\theta}{\partial t^2}\right) \tag{19}$$

Where G is the shear modulus; 2I is the polar moment of inertia; J is the rotational inertial of the cable; θ is the angle of twist (torsion); m_x is a distributed torsional moment per unit length of cable (nonexistent in the present problem), and p_y and p_z are distributed loads in the y and z directions, respectively, per unit length. Lateral motions are coupled to torsional motions through terms of the type $\frac{\partial}{\partial x}\left(\frac{\partial \theta}{\partial x}\frac{\partial y}{\partial x}\right)$ in Eq. 17.

The importance of the coupling can be considered by rewriting the ℓhs of Eq. 17 as

$$2GI\left\{\left(\frac{\partial^2\theta}{\partial x^2}\right)\left[1+\left(\frac{\partial y}{\partial x}\right)^2+\left(\frac{\partial z}{\partial x}\right)^2\right]+\frac{\partial\theta}{\partial x}\frac{\partial y}{\partial x}\frac{\partial^2 y}{\partial x^2}+\frac{\partial\theta}{\partial x}\frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial x^2}\right\}+m_x$$

$$= \ell\text{hs Eq. 17.} \tag{20}$$

The quantities $\partial^2 y/\partial x^2$, $\partial^2 z/\partial x^2$ are essentially proportional to the inverse radii of curvature in the yx- and zx-planes. From an inspection of Fig. 4, the minimum radius of curvature might be expected to be on the order of 1×10^4 feet. With the assumption of a maximum shearing strength of 200,000 psi, the maximum value of the quantity $(\partial\theta/\partial x)$ is given by

$$\frac{\partial\theta}{\partial x} = \frac{\tau_{max}}{G\frac{d}{2}} \approx 12\ \text{ft}^{-1} \tag{21}$$

$$(\text{assuming } G \approx 4 \times 10^6 \text{ psi})$$

The torsional wave velocity in the cable is $(2GI/J)^{1/2} \approx 1 \times 10^3$ ft-sec^{-1}. If a purely sinusoidal torsional wave is considered, the ratio $(\partial\theta/\partial x)\Big/(\partial^2\theta/\partial x^2) = (\lambda/2\pi)$, λ being the wave length.

For

$$\frac{\lambda}{2\pi} \times (1 \times 10^{-4}) \approx \left(\frac{\frac{\partial \theta}{\partial x}}{\frac{\partial^2 \theta}{\partial x^2}} \right) \frac{\partial y}{\partial x} \frac{\partial^2 y}{\partial x^2}$$

to be on the order of unity, therefore, the period of excitation must be about 100 seconds. For such an excitation period, however, the situation is becoming essentially that of static loading. Unless lateral excitations can decrease the radius of curvature greatly from the static values, there would seem to be no effect of this type of excitation upon torsional vibrations.

The effect of torsional vibrations on lateral vibrations is considered by inspection of the magnitude of the term $2GI\,(\partial\theta/\partial x)$ which occurs in Eqs. 18 and 19. This term is a torsional reaction moment and, with the assumed cable properties, is on the order of 3 foot-pounds at maximum shear stress. When the cable loading N is considered to be on the order of 1000 pounds, it is intuitively obvious that the torsional excitations can have no effect on lateral displacements.

The discussion given on torsional-lateral dynamic couplings should not be interpreted to mean that cable twist is not a possible problem caused by rotation of the balloon. We assume, however, that the attachment of the balloon to the cable is such that no transmission of torque is possible from balloon to cable (or from cable to balloon). The analysis of Appendix B (leading to Eq. B-20) shows, in addition, that out-of-plane, steady-state cable configurations, i. e., static three-dimensional cable configurations will not contribute to torsion if no end torque exists at the point of cable attachment.

Coupling between lateral and longitudinal vibration modes will be considered next. With neglect of torsion, the coupled dynamic

equations for two-dimensional cable configurations are (using the coordinate system of Fig. 1)

$$\frac{\partial}{\partial S}\left(T\frac{\partial x}{\partial S}\right) + p_x(t) - mg = m\frac{\partial^2 x}{\partial t^2} \tag{22}$$

$$\frac{\partial}{\partial S}\left(T\frac{\partial y}{\partial S}\right) + p_y(t) = m\frac{\partial^2 y}{\partial t^2} , \tag{23}$$

where $S = S(t)$ is the deformed arc length at time t. The deformed arc length $S(t)$ is related to the arc length S_o of the steady-state deformed cable by the differential expression

$$dS = dS_o + \frac{\partial \eta}{\partial S_o} dS_o = dS_o\left(1 + \frac{\partial \eta}{\partial S_o}\right) \tag{24}$$

where η is the tangential component of the displacement vector (vector representing displacement of a given point of the steady-state shape to its new position at time t by the influence of the dynamic forces).

Now

$$\frac{\partial \eta}{\partial S_o} = \frac{\Delta T}{AE} \tag{25}$$

If $\Delta T = 2 \times 10^3$ lb, $E = 10^7$ psi, and cable diameter = 0.01 ft, $(\partial\eta/\partial S_o) = 0.018$. Therefore, even for a very large increase in cable tension. the approximation $\partial S \approx dS_o$ is valid.

The treatment of the dynamic equations, Eqs. 22 and 23 by the assumption of small departures from the steady-state configuration proceeds as follows.

Let

$$\left.\begin{aligned} x &= x_o + u_c \\ y &= y_o + v_c \end{aligned}\right\} \qquad (26)$$

where u_c, v_c are the coordinates of the displacement vector from the steady-state configuration in the Cartesian system of Fig. 1.

Also let

$$\left.\begin{aligned} T(t) &= T_o + T^t(t) \\ p_x(t) &= p_x^o + p_x^t(t) \\ p_y(t) &= p_y^o(t) + p_y^t(t) \end{aligned}\right\} \qquad (27)$$

With the use of the steady-state conditions (which are equivalent to Eqs. 4),

$$\frac{\partial}{\partial S_o}\left(T_o \frac{\partial x_o}{\partial S_o}\right) + p_x^o - mg = m\frac{\partial^2 x}{\partial t^2} \qquad (28a)$$

$$\frac{\partial}{\partial S_o}\left(T_o \frac{\partial y_o}{\partial S_o}\right) + p_y^o = m\frac{\partial^2 y}{\partial t^2} \qquad (28b)$$

and the approximation $\partial S_o \approx \partial S$, Eqs. 22 and 23 are transformed to

$$T^t \frac{\partial}{\partial S_o}\left(\frac{\partial x_o}{\partial S_o}\right) + \frac{\partial T^t}{\partial S_o}\frac{\partial x_o}{\partial S_o} + T\frac{\partial}{\partial S_o}\left(\frac{\partial u_c}{\partial S_o}\right) + \frac{\partial T}{\partial S_o}\frac{\partial u_c}{\partial S_o} + p_x^t = m\frac{\partial^2 u_c}{\partial t^2} \tag{29}$$

$$T^t \frac{\partial}{\partial S_o}\left(\frac{\partial y_o}{\partial S_o}\right) + \frac{\partial T^t}{\partial S_o}\frac{\partial y_o}{\partial S_o} + T\frac{\partial}{\partial S_o}\left(\frac{\partial v_c}{\partial S_o}\right) + \frac{\partial T}{\partial S_o}\frac{\partial v_c}{\partial S_o} + p_y^t = m\frac{\partial^2 v_c}{\partial t^2} \tag{30}$$

With the assumption that the terms $T^t\ \partial/\partial S_o(\partial x_o/\partial S_o)$, etc., are negligible with respect to $T\ \partial/\partial S_o\ (\partial u/\partial S_o)$, etc., Eqs. 29 and 30 become

$$\frac{\partial T^t}{\partial S_o}\frac{\partial x_o}{\partial S_o} + \frac{\partial}{\partial S_o}\left(T\frac{\partial u_c}{\partial S_o}\right) + p_x^t = m\frac{\partial^2 u_c}{\partial t^2} \tag{31}$$

$$\frac{\partial T^t}{\partial S_o}\frac{\partial y_o}{\partial S_o} + \frac{\partial}{\partial S_o}\left(T\frac{\partial v_c}{\partial S_o}\right) + p_y^t = m\frac{\partial^2 v_c}{\partial t^2} \tag{32}$$

Now, if the Cartesian components of the displacement vector are rotated in such a way that the components are tangential and normal, respectively, to the steady state shape,

$$u_c = \eta\cos\theta - \xi\sin\theta \tag{33a}$$

$$v_c = \xi\cos\theta + \eta\sin\theta \tag{33b}$$

where

$$\cos\theta = \frac{dx_o}{dS_o}, \quad \sin\theta = \frac{dy_o}{dS_o} \tag{33c}$$

Therefore,

$$\frac{\partial T^t}{\partial S_o}\cos\theta + \cos\theta\frac{\partial}{\partial S_o}\left(T\frac{\partial\eta}{\partial S_o}\right) - \sin\theta\frac{\partial}{\partial S_o}\left(T\frac{\partial\xi}{\partial S_o}\right) + p_x^t$$

$$= m\left(\cos\theta\frac{\partial^2\eta}{\partial t^2} - \sin\theta\frac{\partial^2\xi}{\partial t^2}\right) \tag{34a}$$

$$\frac{\partial T^t}{\partial S_o}\sin\theta + \cos\theta\frac{\partial}{\partial S_o}\left(T\frac{\partial\xi}{\partial S_o}\right) + \sin\theta\frac{\partial}{\partial S_o}\left(T\frac{\partial\eta}{\partial S_o}\right) + p_y^t$$

$$= m\left[\cos\theta\frac{\partial^2\xi}{\partial t^2} + \sin\theta\frac{\partial^2\eta}{\partial t^2}\right] \tag{34b}$$

with the assumption that the terms $\partial/\partial S_o(dx_o/dS_o)$, etc., are negligible.

Multiplying Eq. 34a by $-\sin\theta$ and Eq. 34b by $\cos\theta$ and adding the two equations,

$$\frac{\partial}{\partial S_o}\left(T\frac{\partial\xi}{\partial S_o}\right) - p_x^t\sin\theta + p_y^t\cos\theta = m\frac{\partial^2\xi}{\partial t^2} \tag{35}$$

Multiplying Eq. 34a by $\cos\theta$ and Eq. 34b by $\sin\theta$ and adding the two equations,

$$\frac{\partial T^t}{\partial S_o} + \frac{\partial}{\partial S_o}\left(T\frac{\partial\eta}{\partial S_o}\right) + p_x^t\cos\theta + p_y^t\sin\theta = m\frac{\partial^2\eta}{\partial t^2} \tag{36}$$

Equation 35 is the dynamical equation for lateral vibrations; Eq. 36 is the dynamical equation for longitudinal vibrations. Coupling between the two equations occurs through terms depending upon T^t.

Stresses in the cable arise from the force $T(t)$. Accordingly, consideration is given to ways in which the dynamic component T^t could become appreciable. An obvious situation exists when the balloon itself undergoes an erratic movement caused by turbulence--such moments might result in a dramatic increase in cable tension through a longitudinal signal; however, once the balloon is at altitude in the neighborhood of 100,000 feet, the forces exerted on it should be uniform or slowly varying. A more interesting question is concerned with the effect of turbulence on the cable itself. The incidence of a gust on a portion of the cable would cause increased drag and a resultant lateral deflection. If the lateral deflection is large enough, the cable would undergo a local stretching which would then be distributed along the length of the cable by propagation of longitudinal and lateral modes. On the other hand, small lateral deflections would not result in any appreciable increase in cable tension. (In the same way that the vibrations of a stretched string do not cause an appreciable increase in string tension.)

A turbulent gust was modelled in the following way. The maximum amplitude of the gust is taken as 100 ft/sec, varying sinusoidally in time with a period of 3 seconds. The velocity distribution in space is taken as a gaussian exponential form, the width of which at 10 percent of the maximum value (at a given time) is 100 feet. The gust is positioned at 15,500 meters (51,000 feet).

The drag force is assumed to be normal to the cable. Therefore,

$$p_x^t = -p^t \sin\theta \tag{36a}$$

$$p_y^t = p^t \cos\theta \ , \tag{36b}$$

where p^t is the normally exerted drag force.

The drag force is given by the expression

$$p^t = \frac{1}{2}\frac{\rho(x)}{g} C_D \left(\frac{dx_o}{dS_o}\right)^2 D \left\{\left(v_o + v^t - \frac{\partial \xi}{\partial t}\right)\left|\left(v_o + v^t - \frac{\partial \xi}{\partial t}\right)\right| - v_o |v_o|\right\} \tag{37}$$

Where $v_o = v_o(x)$ is the steady state wind profile; $v^t = A \sin \omega t \cdot f(x)$; $A = 100$ ft/sec, $\omega = 2\pi/3$; $\rho(x)$ is the standard atmosphere density; C_D is the drag coefficient calculated as a function of altitude and relative air velocity, and D is the diameter.

The dynamic equation for the lateral deflection of the cable is, therefore,

$$\frac{\partial}{\partial S}\left(T \frac{\partial \xi}{\partial S}\right) + p^t = m \frac{\partial^2 \xi}{\partial t^2} \quad . \tag{38}$$

With the assumption that $T(t) = T_o$, the dynamics program presented in Appendix C, was applied to obtain the lateral deflection ξ as a function of time, starting the application of the gust velocity $A \sin \omega t$ at time $t = 0$. The deflections at the excitation midpoint (15,000 meters altitude) for approximately 3 cycles are shown in Fig. 5. Because the dynamics program requires the solution of a matrix with dimensions equal to the number of mass points chosen along the length of the cable and because the spacing of the mass points was taken as 6 meters in order to assure accuracy, it was found that the computer employed, the CDC 3600, was not large enough to consider the motion of all points on the cable. This difficulty was overcome by the use of the artificial end conditions that the lateral deflections from the static configuration

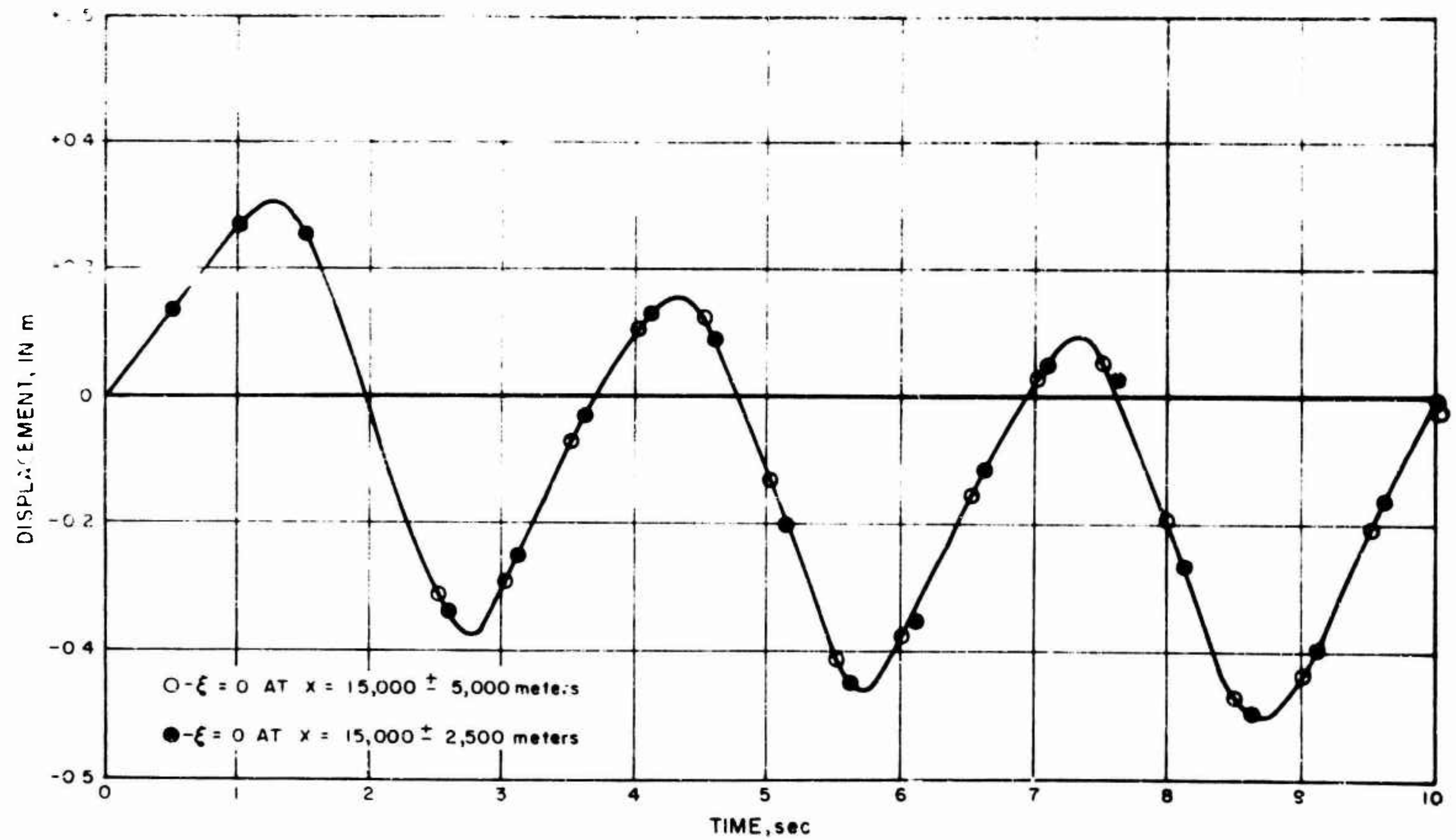

Figure 5
Displacement of mid-point of gust excitation (at 15,000 meters altitude) as a function of time

PA-3-10233

were exactly zero at distances well removed from the point of excitation. The points of the curve given in Fig. 5 were obtained with two sets of end conditions,

a) at $x = 15,000 \pm 2,500$ meters

$\xi = 0$

b) at $x = 15,000 \pm 5,000$ meters

$\xi = 0$

Both sets of end conditions gave identical results, indicating that the motion caused by a localized gust at 15,000 meters was damped out at both sets of end points. The lateral wave velocity at the 15,000-meter altitude point was approximately 1000 m/sec; with the relationship between arc length and altitude given by the static configuration of Fig. 4, it is found that a 9-second interval is sufficient so that reflected waves from the end points at $15,000 \pm 2,500$ meters altitude have reached the midpoint.

In order to demonstrate the results of the dynamics program more dramatically, the form of the lateral wave for various points in time is shown in Fig. 6 with the end conditions $\xi = 0$ at $x = 15,000 \pm 2,500$ meters.

Because gusts are statistical and do not recur with any fixed period, the use of the sinusoidal excitation term is somewhat artificial. The dynamics program is, however, perfectly adapted to the use of a transient excitation. In the example chosen here the first 3 seconds or, indeed, the first 1.5 seconds of behavior could be taken as representative of a gust. It is believed that the gust amplitude and period are conservative.

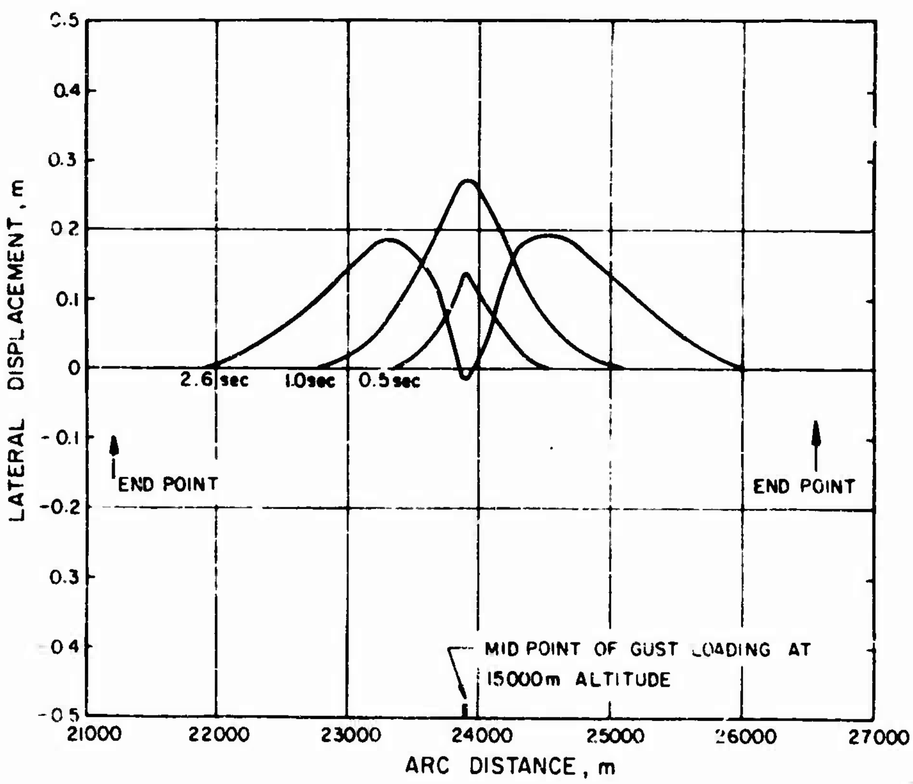

Figure 6
Lateral displacement along cable for three times after initial gust excitation

PA-3-10234

The displacements shown in Fig. 5 are of the order of 1 meter at best. With steady state radii of curvature of the order of 10^4 meters, a 1-meter displacement will not significantly change the cable tension. For example, assume that the cable length increases by $\left\{\left(10^4 + 1\right) - 10^4\right\}/10^4 = 10^{-4}$, as a fraction of the original (steady state) arc length at a point of 1-meter cable displacement. With the cable properties employed, the cable tension (load) would increase by $10^{-4} \times 3.451 \times 10^4 \approx 3$ Kgf, whereas the steady-state tension is approximately 800 Kgf.

It is concluded, therefore, that the effects of gust loading on a tether cable already heavily loaded does not seriously affect cable tension.

5. CABLE LOADING FROM VORTEX SHEDDING

In the project quarterly report (Ref. 4), the statement was made that vortex shedding loads were not considered to be important. A further analysis of the problem was made and is presented in Appendix D. Previous conclusions that vortex shedding loads are not important were verified.

6. CONCLUSIONS CONCERNING POSSIBLE CABLE LOADING PROBLEMS

With the use of a typical, steady-state wind velocity profile (in two dimensions), it has not been found that interactions between torsional, longitudinal and lateral, vibrations caused by unsteady wind loading conditions are important. Indeed, unsteady wind conditions along the cable have not been found to cause important dynamic loadings by excitation of any one particular mode. It would appear that balloon motions are much more important than are direct cable loadings in causing cable stress changes. It has been shown that the cable is weak in torsion, and certainly no coupling should be allowed to exist between the cable and the balloon that would permit a rotating balloon to exert torque on the cable.

It is entirely possible that balloon motions and the weakness of the cable material in compression might give rise to destructive effects in the longitudinal mode. This may occur when a downward motion of the balloon is suddenly induced during ascent, when the length of tether is short. The cable, normally under heavy tension loading, is momentarily relieved at its upper end and an unloading (compressive) signal is propagated along the cable length. This compressive signal should travel with essentially undiminished amplitude until reflection occurs at some point of cable attachment, e.g., at its mooring point. Now reflection of a compressive stress from a rigid attachment leads to a doubling of compressive stress and, hence, to the existence of a net compressive stress in the cable. Such a compressive stress (equal in magnitude to the original tension stress) might well be very damaging. On the other hand, the effective point of cable attachment may not be rigid; for example, termination at a partially wound drum, and the reflection of the compressive signal might be greatly lessened in severity.

7. RECOMMENDED LABORATORY AND FIELD TESTING OF CABLE DYNAMIC EFFECTS

The following tests are recommended.

a) Laboratory tests on the effects of the sudden reduction of tension loading of the glass fiber-resin cable near anchor points of differing types. Anchoring configurations that should be studied include

1) a solid, vise clamp
2) a partially wound drum
3) a mass of appreciable inertia solidly clamped to the cable between the anchor and the free end

b) High altitude balloon field tests should be conducted with special emphasis on cable motions. Measurements of cable motions could be made by suitably mounted accelerometers at intervals along the cable with data telemetered by radio transmitters.

Although the load of the transmitters should be imperceptible on the cable, this load can be reduced by supporting the transmitters on small balloons. In this case, drag from the balloons would also be imposed on the cable.

The additional balloons would also allow measurements of wind speed, wind direction, and atmospheric conditions at desired intervals. Thus, motions of the cable in known wind fields could be correlated.

The overall configuration of the cable could be made visible by attaching markers such as flags at regular intervals

along the cable as it is payed out during ascension. The string could be photographed from a distance or visual observations could be made by telescopes mounted for triangulation.

Cable tension at the balloon and at the ground as well as the cable angle at these two positions would, of course, be important

8. SUMMARY AND CONCLUSIONS

A computer study has been made of nonsteady aerodynamic loadings on a long cable of the continuous glass fiber-resin type used as a tether for a balloon at altitudes of approximately 100,000 feet. No important interactions between torsional, longitudinal, and lateral modes were found. Furthermore, the effects of lateral loadings from gusts or vortex sheddings were found to be unimportant. Computer programs are presented that enable computations to be made of cable motions resulting from localized gust loadings and from vortex shedding phenomena. Theoretical results obtained to date indicate that the high strength-to-weight ratios obtainable with the continuous glass fiber-resin cables will lead to an effective tether for high altitude balloons.

The present dynamic study has been concerned with cable behavior in the fully extended configuration. It is believed that benefits can be obtained by extending the study of system dynamics to include the motion of the balloon and cable during launch, the period in which the balloon rises to its maximum altitude, and recovery operations. Such a study should also delineate the effects of a streamlined cross sectional cable shape and other techniques for reducing aerodynamic drag. Such a reduction in drag might have an effect on steady-state configurations and balloon behavior during the rise period. A study of this type might also better define the way in which the cable should be payed out during the rise period (for a given wind profile) and would thus enable wind performance to be specified more effectively.

Certain laboratory and field tests are recommended for further studies of the effectiveness of the continuous glass fiber-resin cable as a balloon tether.

9. REFERENCES

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5. Phillips, O. M., "The Intensity of Aeolian Tones," Journ. of Fluid Mechanics, Vol. 1, 1956, pp. 607 - 624

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APPENDIX A

STEADY-STATE CABLE PROFILE PROGRAM

Notation

D	diameter of cable (meters)
EA	force (Kgf^2)
AMO	m_o = initial mass/unit length (Kg/m)
W	weight of balloon (Kgf)
G	gravity at sea level (m/sec^2)
DIST	ℓ = length of cable (meters)
XL	x'_ℓ = initial altitude (meters)
YL	y'_ℓ = initial displacement (meters)
EP	ϵ = tolerance defining the convergence of the sequence of altitudes computed at the lower end of the cable (meters)
DSI	ΔS = arc length step size (meters)
PN	number of intervals desired in final integration
PNP	printing interval (i. e., print every PNP step)
DSB	diameter of balloon (meters)
PITER	maximum number of iterations performed to obtain convergence
OPTION	a flag such that: 1 = spacial history of cable is printed after each iteration 0 = omits this printout
TAP	a flag such that: 1 = cable information is written on tape for use in dynamics program 0 = omits writing information on tape

Required Subroutines

RUNGS	4th order R-K integration routine for 1st order system
DERIVE	evaluates 1st derivatives
EROR	gives error code and aborts program
RHOX	computes mass density versus altitude
NU	computes kinematic viscosity versus altitude
CDR	computes drag coefficient versus Reynolds number for cable
CDBAR	computes drag coefficient versus Reynolds number for balloon
INTER	uses linear interpolation to obtain wind velocity versus altitude from a given table

Required Data

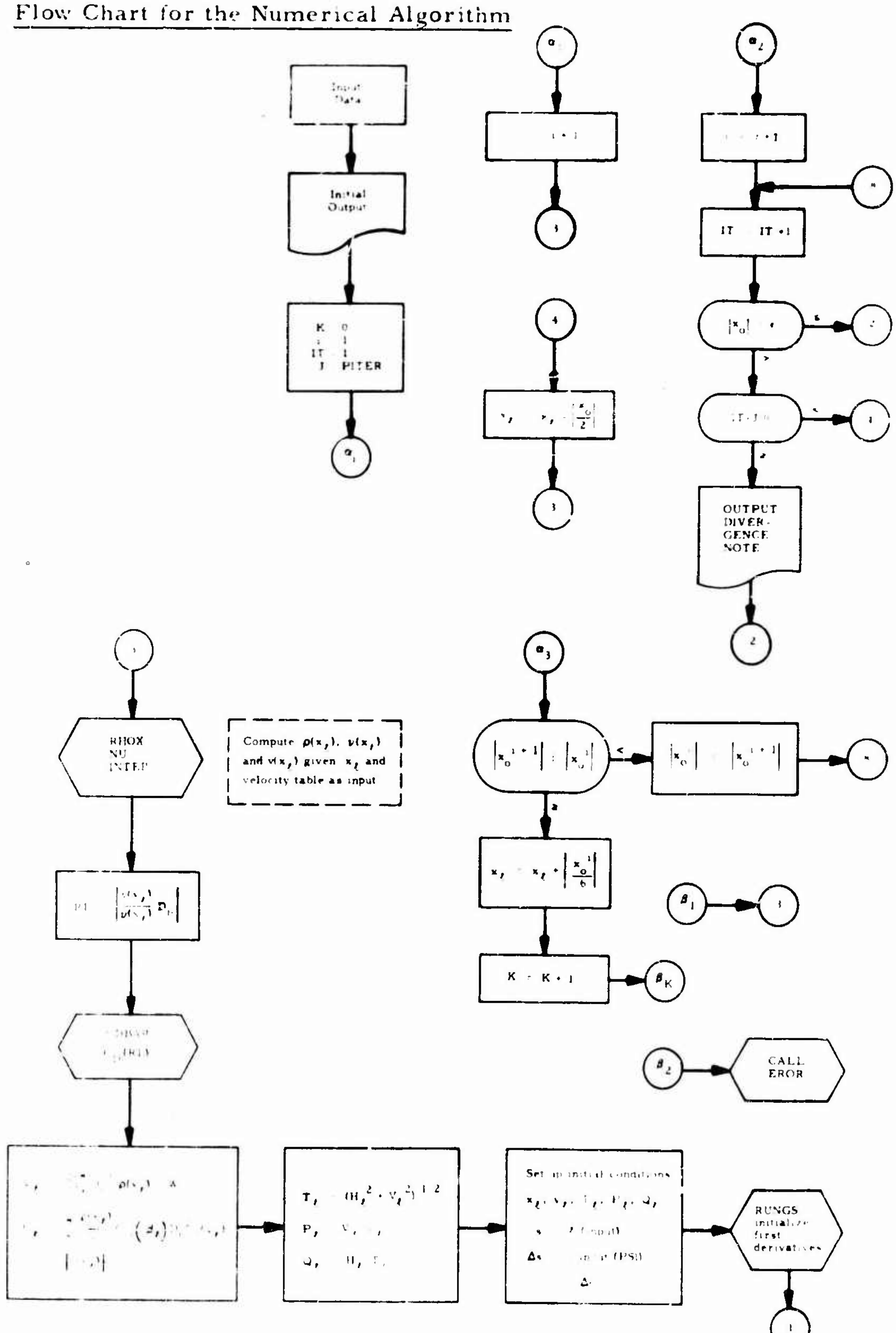
FORTRAN Quantity	Math Symbol	Units	Test Case
D	D	m	0.0025
EA	EA	Kg f	3.451×10^{4}
AMO	m_o	Kg/m	8.0×10^{-4}
W	W	Kgf	50.0
G	g	m/sec^2	9.81
DIST	ℓ	m	36000.0
XL	x_ℓ^1	m	28000.0
YL	y_ℓ	m	20000.0

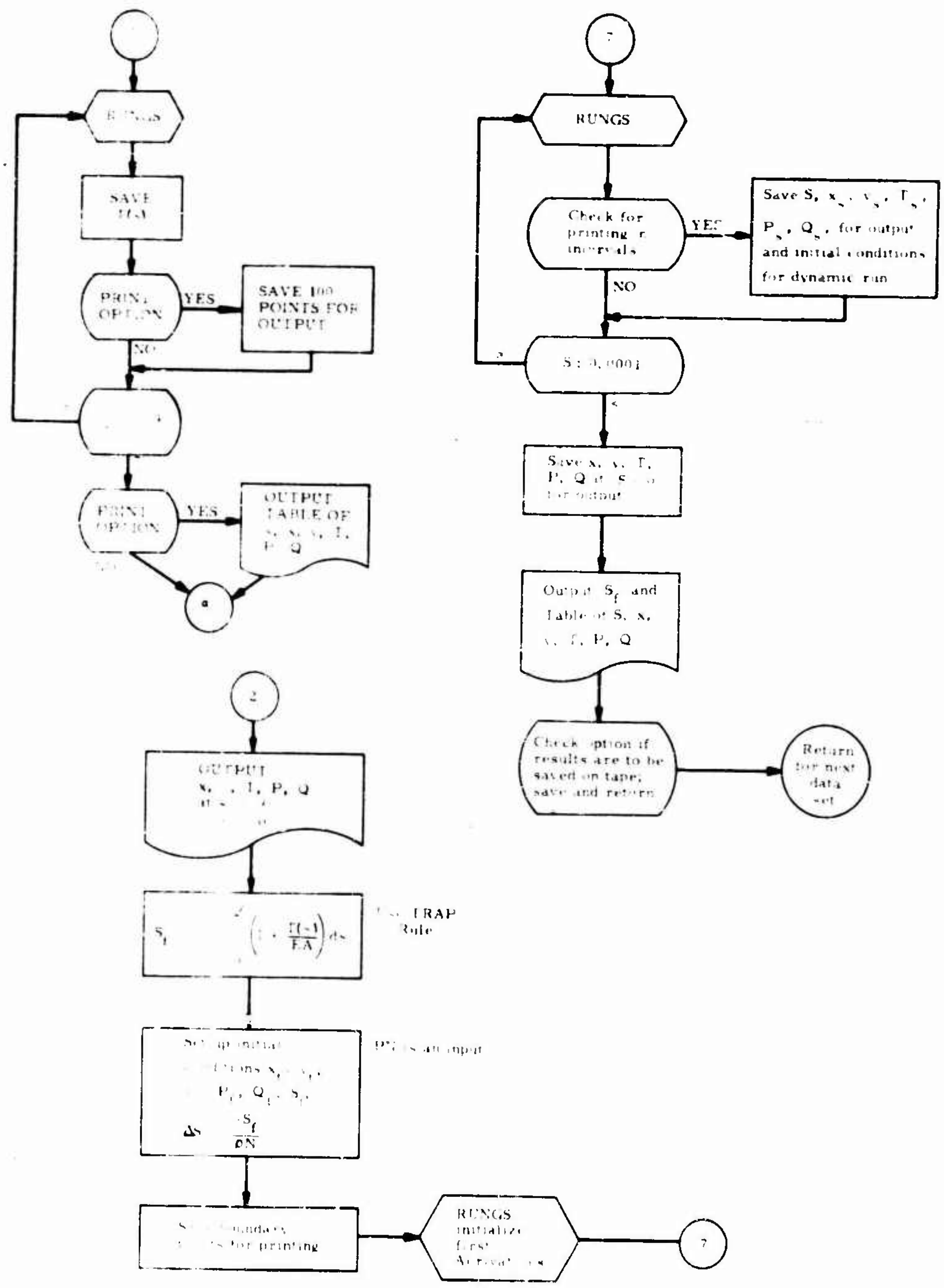
Required Data (continued)

FORTRAN Quantity	Math Symbol	Units	Test Case
DL			Not used-leave field blank
EP	ϵ	m	10.0
DSI	ΔS	m	6.0
PN	PN		6000.0
PNP			10.0
PSB	D_b	m	40.0
PITER			6.0
OPTION			1.0
TAP			1.0

Table of x vs. v (x)

Flow Chart for the Numerical Algorithm





SAVE 100 POINTS FOR OUTPUT
YES
NO
PRINT OPTION
YES
OUTPUT TABLE OF
RUNGS
Check for printing intervals
YES
Save S, x_s, y_s, T_s, P_s, Q_s, for output and initial conditions for dynamic run
NO
Output S_f and Table of S, x, y, T, P, Q
Check option if results are to be saved on tape; save and return
Return for next data set
OUTPUT
TRAP Rule
RUNGS initialize first

```
      PROGRAM BALLOON
C     COMPUTES STATIC DEFLECTION - UNITS IN M-KG-SEC
      DIMENSION SS(1000),XS(1000),YS(1000),PS(1000),GS(1000),Y(5),YP(5),
     1HEAD(9),V(100),XV(100),TSS(10005),TS(1000),XSS(10005),PSS(10005)
      COMMON N,NS,ENT,V,XV,D,AMO,EA,G,DSB
      COMMON /1/ XSS,SS,XS,YS
 1000 READ (60,3) (HEAD(I),I=1,9)
    3 FORMAT(9A8)
      IF (HEAD(1)) 4,5,4
    5 CALL EXIT
    4 READ (60,1) D,EA,AMO,W,G,DIST,XL,YL,DL,EP,DSI,PN,PNP,DSB,PITER
     1,OPTION,TAP
    1 FORMAT (5E14.6)
      NNP=PNP
      READ (60,2) N,(XV(I),V(I),I=1,N)
    2 FORMAT (I15/(6E12.6))
      WRITE (6 ,3) (HEAD(I),I=1,9)
      WRITE (6 ,9) D,EA,AMO,W,G,DIST,XL,YL,DL,  EP,DSI,PN,DSB
    9 FORMAT(/5H D = ,1E15.5,7H  EA = ,1E15.5,7H  MO = ,1E15.5,6H  W = ,
     11E15.5,6H  G = ,1E15.5//8H DIST = ,1E15.5,7H  XL = ,1E15.5,7H  YL
     2= ,1E15.5,7H  DL = ,1E15.5           //6H EP = ,1E15.5,8H  DS
     3I = ,1E15.5,7H  PN = ,1E15.5,6H  DSB = ,1E15.5)
      LOGIC= 1
      DPR=DIST/100.
      DL=DL
      KL=0
      ITERAT=1
      JT=PITER
      NNNP=NNP
   50 GO TO (11,13,14),LOGIC
   11 LOGIC=LOGIC+1
    8 CALL RHOX (XL,RHO)
      CALL NU (XL,GNU)
      NS=1
      CALL INTER (XL,V,XV,SVL)
      RL=ABS(SVL*DSB/GNU)
      CALL CDBAR (RL,CDB)
        CVL=.451378254*RHO*DSB**3-W
      HL=.55269081*RHO*CDB*DSB**2*SVL*ABS(SVL)/G
      TL=SQRT(HL**2+CVL**2)
      PL=CVL/TL
      GL=HL/TL
      Y(1)=XL
      Y(2)=YL
      Y(3)=TL
      Y(4)=PL
      Y(5)=GL
      T=0.
      I=1
      S=DIST
      DS=-DSI
      ID=0
      CALL RUNGS (S,DS,5,Y,YP,ID)
      TSS(I)=Y(3)
      SP=S-DPR
      J=1
      SS(J)=S
      XS(J)= Y(1)
      YS(J)= Y(2)
      TS(J)= Y(3)
      PS(J)= Y(4)
```

```
      QS(J)= Y(5)
    6 CALL RUNGS (S,DS,5,Y,YP,ID)
      I=I+1
      TSS(I)=Y(3)
      IF (OPTION) 91,92,91
   91 IF (DIST-S-DPRS)  92,95,95
   95 DPRS=DPRS+DPR
      J=J+1
      SS(J)=S
      XS(J)= Y(1)
      YS(J)= Y(2)
      TS(J)= Y(3)
      PS(J)= Y(4)
      QS(J)= Y(5)
   92 IF (S-.0001)  7,7,6
    7 IF (OPTION) 94,50,94
   94 JJ=J+1
      SS(JJ) = S
      XS(JJ) = Y(1)
      YS(JJ) = Y(2)
      TS(JJ) = Y(3)
      PS(JJ) = Y(4)
      QS(JJ) = Y(5)
      WRITE (6 ,18) (SS(II),XS(II),YS(II),TS(II),PS(II),QS(II),II=1,JJ)
      GO TO 50
   13 LOGIC=LOGIC+1
      Y1=ABS(Y(1))
   24 ITERAT=ITERAT+1
      IF (ABS(Y(1))-EP) 20,20,86
   86 IF (ITERAT-JT) 26,87,87
   26 XL=XL-Y(1)/2.
      GO TO 8
   14 IF (ABS(Y(1))-Y1) 15,16,16
   15 Y1=ABS(Y(1))
      GO TO 24
   16 XL=XL+Y1/6.
      KL=KL+1
      GO TO (8,54),KL
   54 CALL ERUR(1)
   87 WRITE (6 ,88)
   88 FORMAT (11H1DIVERGENCE)
   20 WRITE (6 ,18) DIST,XL,YL,TL,PL,QL,S,(Y(J),J=1,5)
   18 FORMAT (///9X,1HS,18X,1HX,18X,1HY,18X,1HT,18X,1HP,18X,1HQ//
     1(6E19.7))
      SUM=1.+(TSS(1)+TSS(I))/(2.*EA)
      II = I-1
      DO 40 J=2,II
   40 SUM=SUM+(1.+TSS(J)/EA)
      SUM = SUM * DSI
      WRITE (6 ,65) SUM
   65 FORMAT (/// 16H TOTAL LENGTH = ,1E18.6)
      YL = YL -Y(2)
      S = SUM
      DS = -SUM/PN
      NS=1
      I =1
      J =1
      Y(1)=XL
      Y(2)=YL
      Y(3)=TL
      Y(4)=PL
```

```
      Y(5)=QL
      SS(1) =S
      XS(1) =XL
      YS(1) =YL
      TS(1) =TL
      PS(1) =PL
      QS(1) =QL
      ENT=1.
      ID=0
      XSS(1)=Y(1)
      TSS(1)=Y(3)
      PSS(1)=Y(4)
      CALL RUNGS (S,DS,5,Y,YP,ID)
   60 CALL RUNGS (S,DS,5,Y,YP,ID)
      I=I+1
      XSS(I)=Y(1)
      TSS(I)=Y(3)
      PSS(I)=Y(4)
      IF (I -NNNP) 61,62,62
   62 NNNP=NNNP+NNP
      J=J+1
      SS(J)=S
      XS(J)= Y(1)
      YS(J)= Y(2)
      TS(J)= Y(3)
      PS(J)= Y(4)
      QS(J)= Y(5)
   61 IF (S-.0001) 64,64,60
   64 JJ = J+1
      SS(JJ) = S
      XS(JJ) = Y(1)
      YS(JJ) = Y(2)
      TS(JJ) = Y(3)
      PS(JJ) = Y(4)
      QS(JJ) = Y(5)
      WRITE (6 ,16)  (SS(L),XS(L),YS(L),TS(L),PS(L),QS(L),L=1,JJ)
      IF (TAP) 163,1000,163
  163 WRITE (6,8000)I,DSI,DIST
 8000 FORMAT(1H1,I9,25H POINTS AT A INCREMENT OF,E15.8,12H STARTING AT,
     1E15.8)
      WRITE (2) (XSS(L),L=1,I)
      WRITE (2) (TSS(L),L=1,I)
      WRITE (2) (PSS(L),L=1,I)
      GO TO 1000
      END
      SUBROUTINE RHOX(X,RHO)
C     MASS DENSITY VS ALTITUDE
      RHO=.25-X/16000.
      RHO=10.**RHO
      RETURN
      END
      SUBROUTINE NU (X,GNU)
C     KINEMATIC VISCOSITY VS ALTITUDE
      IF (X-10000.) 1,1,2
    1 GNU=-4.623+X/26800.
      GO TO 5
    2 IF (X-17700.) 3,3,4
    3 GNU=X/17100.-5.035
      GO TO 5
    4 GNU=X/14250.-5.2421
    5 GNU=10.**GNU
```

```
      RETURN
      END
      SUBROUTINE INTER (XL,V,XV,SVL)
C     GIVEN THE ALTITUDE, THIS ROUTINE INTERPOLATES LINEARLY TO
C     OBTAIN THE WIND VELOCITY FROM INPUT TABLE
      DIMENSION  V(100),XV(100)
      COMMON N,NS
      NN=N-NS+1
      IF (XL-XV(NN)) 5,5,6
    6 NS=1
    5 DO 1 I=NS,N
      J=N-I+1
      IF (XL-XV(J)) 1,3,4
    3 SVL=V(J)
      GO TO 2
    4 SVL=(V(J+1)-V(J))*(XL-XV(J+1))/(XV(J+1)-XV(J))+V(J+1)
      GO TO 2
    1 CONTINUE
    2 NS=J+1
      RETURN
      END
      SUBROUTINE CDBAR (RL,CDB)
C     DRAG COEFFICIENT VS REYNOLDS NUMBER FOR BALLOON
      IF (RL-1.0) 1,1,2
    1 CDB=27.4*RL**(.961)
      GO TO 10
    2 IF (RL-10.) 3,3,4
    3 CDB=27.4*RL**(-.604)
      GO TO 10
    4 IF (RL-100.) 5,5,6
    5 CDB=16.0*RL**(-.572)
      GO TO 10
    6 IF (RL-1130.) 7,7,8
    7 CDB=7.25*RL**(-.4)
      GO TO 10
    8 IF (RL-30000.) 9,9,11
    9 CDB=.4
      GO TO 10
   11 CDB=.44
   10 RETURN
      END
      SUBROUTINE  RUNGS (X,H,N,Y,YPRIME,INDEX)
C     X    INDEPENDENT VARIABLE
C     H    INCREMENT DELTA X, MAY BE CHANGED IN VALUE
C     N    NUMBER OF EQUATIONS
C     Y    DEPENDENT VARIABLE BLOCK      ONE DIMENSIONAL ARRAY
C     YPRIME  DERIVATIVE BLOCK  ONE DIMENSIONAL ARRAY
C     THE PROGRAMMER MUST SUPPLY INITIAL VALUES OF  Y(1) TO Y(N)
C     INDEX  IS A VARIABLE WHICH SHOULD BE SET TO ZERO BEFORE EACH
C     INITIAL ENTRY TO THE SUBROUTINE, I.E., TO SOLVE A DIFFERENT
C     SET OF EQUATIONS OR TO START WITH NEW INITIAL CONDITIONS.
C     THE PROGRAMMER MUST WRITE A SUBROUTINE CALLED DERIVE WHICH COM-
C     PUTES THE DERIVATIVES AND STORES THEM
C     THE ARGUMENT LIST IS   SUBROUTINE DERIVE(X,N,Y,YPRIME)
      DIMENSION Y(5),YPRIME(5),W4(5),Z(5),W1(5),W2(5),W3(5)
      IF (INDEX) 5,5,1
    1 DO 2 I=1,N
      W1(I)=H*YPRIME(I)
    2 Z(I)=Y(I)+(W1(I)*.5)
      A=X+H/2.
      CALL DERIVE(A,N,Z,YPRIME)
```

```
      DO 3 I=1,N
      W2(I)=H*YPRIME(I)
    3 Z(I)=Y(I)+.5*W2(I)
      A=X+H/2.
      CALL DERIVE(A,N,Z,YPRIME)
      DO 4 I=1,N
      W3(I)=H*YPRIME(I)
    4 Z(I)=Y(I)+W3(I)
      A=X+H
      CALL DERIVE (A,N,Z,YPRIME)
      DO 7 I=1,N
      W4(I)=H*YPRIME(I)
    7 Y(I)=Y(I)+(((2.*(W2(I)+W3(I)))+W1(I)+W4(I))/6.)
      X=X+H
      CALL DERIVE (X,N,Y,YPRIME)
      GO TO 6
    5 CALL DERIVE (X,N,Y,YPRIME)
      INDEX=1
    6 RETURN
      END
      SUBROUTINE DERIVE (X,NN,Y,YP)
      DIMENSION V(100),XV(100),Y(5),YP(5)
      COMMON N,NS,ENT,V,XV,D,AMO,EA,G,DSB
      Z =Y(1)
      CALL RHOX (Z,RHO)
      CALL NU (Z,GNU)
      CALL INTER (Z,V,XV,SVL)
      R=ABS(SVL*D*Y(4)/GNU)
      CALL CDR (R,CD)
      PN =.5*RHO*CD*Y(4)**2*D*SVL*ABS(SVL)/G
      PX = PN*Y(5)
      PY = -PN*Y(4)
      IF (ENT) 1,2,1
    2 DEM= 1.+ Y(3)/EA
      AM=AMO/DEM
      PGM=PX +AM*G
      YP(1)=Y(4)*DEM
      YP(2)= Y(5)*DEM
      YP(3)= (PGM*Y(4)+PY*Y(5))*DEM
      YP(4) =YP(2)*(PGM*Y(5) -PY* Y(4))/Y(3)
      YP(5)= YP(1)*(PY*Y(4)-PGM*Y(5))/Y(3)
      RETURN
    1 PGM= PX +AMO*G
      YP (1) = Y(4)
      YP(2) = Y(5)
      YP(3) = PGM*Y(4)+PY*Y(5)
      YP(4) = (PGM*Y(5)-PY*Y(4))*Y(5)/ Y(3)
      YP(5) = (PY*Y(4)-PGM*Y(5))*Y(4)/Y(3)
      RETURN
      END
      SUBROUTINE CDR(R,CD)
C     DRAG COEFFICIENT VS REYNOLDS NUMBER FOR CABLE
      IF (R-2.23) 1,1,2
    1 CD=10.8*R**(-.742)
      GO TO 10
    2 IF (R-8.0) 3,3,4
    3 CD=9.15*R**(-.526)
      GO TO 10
    4 IF (R-1000.0) 5,5,6
    5 CD=4.95*R**(-.232)
      GO TO 10
```

```
6 IF (R-10000.) 7,7,8
7 CD=1.0
  GO TO 10
8 CD=1.15
10 RETURN
  END
  SUBROUTINE EROR (I)
  I=I
  WRITE (61,1) I
1 FORMAT (13H ERROR CODE =,I15)
  CALL EXIT
  RETURN
  END
```

APPENDIX B

SIGNIFICANCE OF TORSION

by Dr. S. Fersht

Consider a three-dimensional right handed Cartesian system of co-ordinates (x, y, z), fixed at the top of the riser with x (vertical) directed downwards. In each horizontal section of the riser we have three forces and three moments in the direction of the axis (Fig. B-1).

Considering an element of the riser there are, in addition to the internal forces and moments, external loads and inertia forces in the y, z directions. Finally, a distributed external moment m_x will be considered. Coupling of motion in the "x" direction with "y", "z" motion is a separate problem which is not considered here, so there is no equation for "x" motion.

Deflections due to shear, and rotational inertia about axes normal to the deflected shape of the riser, are factors which need be considered only when there is reason to expect deformations in modes whose wave lengths are in the neighborhood of the diameter of the riser. For the riser, major exciting forces with periods lower than 0.003 seconds would be required before the inclusion of the shear and rotary inertia would be warranted, and those complicating factors are therefore neglected.

The remaining equations of motion for the elemental Fig. B-1 are

$$\frac{\partial N}{\partial x} + p_x = 0 \qquad \frac{\partial S_y}{\partial x} + p_y = m\frac{\partial^2 y}{\partial t^2} \qquad \frac{\partial S_z}{\partial x} + p_z = m\frac{\partial^2 t}{\partial t^2} \tag{B-1}$$

$$\frac{\partial M_x}{\partial x} + S_z\frac{\partial y}{\partial x} - S_y\frac{\partial z}{\partial x} + m_x = J\frac{\partial^2 \theta}{\partial t^2} \tag{B-2}$$

$$\frac{\partial M_y}{\partial x} + N\frac{\partial z}{\partial x} - S_z = I\frac{\partial^2 \theta}{\partial t^2}\frac{\partial y}{\partial x} \tag{B-3}$$

$$\frac{\partial M_z}{\partial x} - N\frac{\partial y}{\partial x} + S_y = I\frac{\partial^2 \theta}{\partial t^2}\frac{\partial z}{\partial x} \tag{B-4}$$

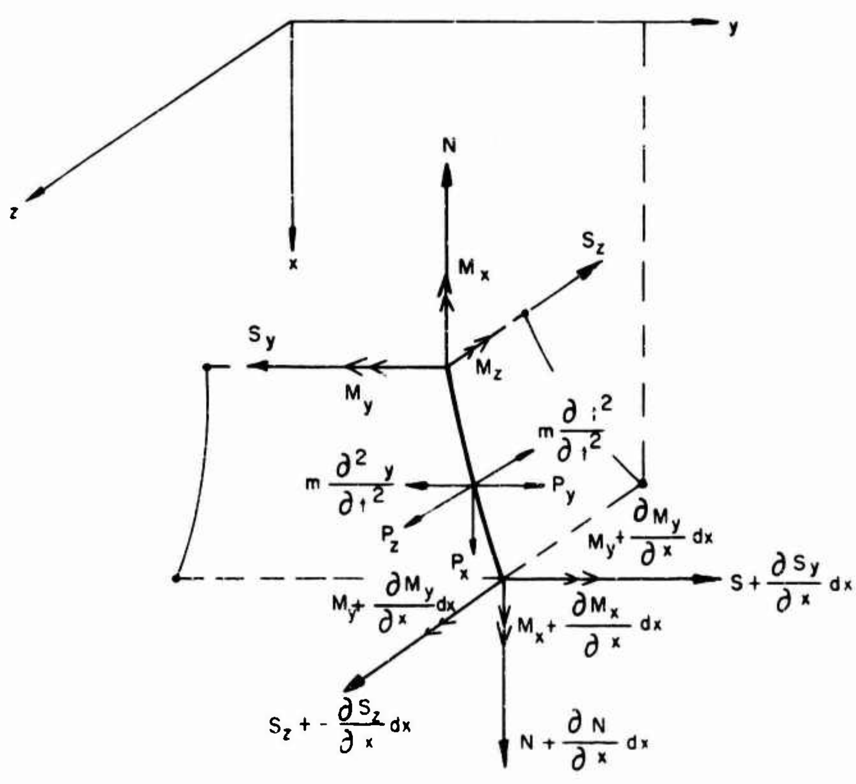


Figure B-1

PA-3-8116

Eliminating S_y and S_z from Eqs. B-2, B-3, and B-4,

$$\frac{\partial M_x}{\partial x} + \frac{\partial y}{\partial x}\frac{\partial M_y}{\partial x} + \frac{\partial z}{\partial x}\frac{\partial M_z}{\partial x} = J\left[1 + \left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial x}\right)^2\right]\frac{\partial^2 \theta}{\partial t^2} \qquad \text{(B-5)}$$

Since $\left(\frac{\partial y}{\partial x}\right)^2 \ll 1$ $\left(\frac{\partial z}{\partial x}\right)^2 \ll 1$, the last equation can be written

$$\frac{\partial M_x}{\partial x} + \frac{\partial y}{\partial x}\frac{\partial M_y}{\partial x} + \frac{\partial z}{\partial x}\frac{\partial M_z}{\partial x} = J\frac{\partial^2 \theta}{\partial t^2} \qquad \text{(B-5')}$$

We now find the stress strain relations for the beam. In order to understand the geometry of deformation of such a beam, and derive the stress strain relations, we have to use some of the concepts of a space curve. In our case the problem is much simplified by the fact that the cross section of the riser has circular symmetry. This permits each point along the axis of the riser in the deformed shape to be described by a radius vector

$$\overline{r} = x\,\overline{i} + y\,\overline{j} + z\,\overline{k} \qquad \text{(B-6)}$$

where $\overline{i}$, $\overline{j}$ and $\overline{k}$, are unit vectors along the axes of the coordinate system. Assuming that

$$|\overline{dr}| = ds \approx dx$$

the unit vector tangent to the deformed riser is

$$\overline{t} = \frac{\overline{dr}}{ds} \approx \overline{i} + \frac{\partial y}{\partial x}\,\overline{j} + \frac{\partial z}{\partial x}\,\overline{k} \qquad \text{(B-7)}$$

The plane in which the riser lies at each point is the osculating plane. This plane is determined by two vectors; the vector $\bar{t}$ and, a unit vector $\bar{n}$ which is normal to the riser. It is known from differential geometry that

$$\frac{\partial \bar{t}}{\partial S} \approx \frac{\partial \bar{t}}{\partial x} = \frac{1}{\rho}\,\bar{n} = \frac{\partial^2 y}{\partial x^2}\,\bar{j} + \frac{\partial^2 z}{\partial x^2}\,\bar{k} \tag{B-8}$$

where ρ is the curvature of the deformed riser, measured in the oscillating plane. The unit vector normal to $\bar{t}$ and $\bar{n}$, i.e., the unit vector normal to the osculating plane is,

$$\bar{b} = \bar{t} \times \bar{n} = \rho\left[\left(\frac{\partial y}{\partial x}\,\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial x}\,\frac{\partial^2 y}{\partial x^2}\right)\bar{i} - \frac{\partial^2 z}{\partial x^2}\,\bar{j} + \frac{\partial^2 y}{\partial x^2}\,\bar{k}\right] \tag{B-9}$$

Elastic relationships for bending and torsion are

$$\frac{1}{\rho} = \frac{M_b}{EI}\,; \qquad \frac{\partial \theta}{\partial x} = \frac{T}{2GI} \tag{B-10}$$

By ordinary sign con[illegible]ions, the bending moment vector for a cross section with an outer [illegible] normal $\bar{t}$, is in the direction of $\bar{b}$, while the torsion moment vector is in the direction of $\bar{t}$. Using Eqs. B-7, B-8, B-9 and B-10, one can write

$$M_b\,\bar{b} = \frac{EI}{\rho}\,\bar{b} = EI\left(\frac{\partial y}{\partial x}\,\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial x}\,\frac{\partial^2 y}{\partial x^2}\right)\bar{i} - EI\,\frac{\partial^2 z}{\partial x^2}\,\bar{j} + EI\,\frac{\partial^2 y}{\partial x^2}\,\bar{k} \tag{B-11}$$

$$T\,\bar{t} = 2\,GI\,\frac{\partial \theta}{\partial x}\,\bar{t} = T\,\bar{i} + T\,\frac{\partial y}{\partial x}\,\bar{j} + T\,\frac{\partial z}{\partial x}\,\bar{k} \tag{B-12}$$

The sum of the components of these two vectors along the axes are,

$$M_x = T + EI\left(\frac{\partial y}{\partial x}\,\frac{\partial^2 z}{\partial x^2} - \frac{\partial z}{\partial x}\,\frac{\partial^2 y}{\partial x^2}\right)$$

$$M_y = T\frac{\partial y}{\partial x} - EI\frac{\partial^2 z}{\partial x^2} \tag{B-13}$$

$$M_z = T\frac{\partial z}{\partial x} + EI\frac{\partial^2 y}{\partial x^2}$$

Using Eqs. B-1, B-3, B-4, B-5, B-10 and B-13, one obtains

$$2G\left[\frac{\partial}{\partial x}\left(I\frac{\partial\theta}{\partial x}\right) + \frac{\partial y}{\partial x}\frac{\partial}{\partial x}\left(I\frac{\partial\theta}{\partial x}\frac{\partial y}{\partial x}\right) + \frac{\partial z}{\partial x}\frac{\partial}{\partial x}\left(I\frac{\partial\theta}{\partial x}\frac{\partial z}{\partial x}\right)\right] + m_x = J\frac{\partial^2\theta}{\partial t^2} \tag{B-14}$$

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right) + 2\frac{\partial^2}{\partial x^2}\left(GI\frac{\partial\theta}{\partial x}\frac{\partial z}{\partial x}\right) - \frac{\partial}{\partial x}\left(N\frac{\partial y}{\partial x}\right) - p_y = -m\frac{\partial^2 y}{\partial t^2} + \frac{\partial}{\partial x}\left(J\frac{\partial z}{\partial x}\frac{\partial^2\theta}{\partial t^2}\right) \tag{B-15}$$

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 z}{\partial x^2}\right) - 2\frac{\partial^2}{\partial x^2}\left(GI\frac{\partial\theta}{\partial x}\frac{\partial y}{\partial x}\right) - \frac{\partial}{\partial x}\left(N\frac{\partial z}{\partial x}\right) - p_z = -m\frac{\partial^2 z}{\partial t^2} - \frac{\partial}{\partial x}\left(J\frac{\partial y}{\partial x}\frac{\partial^2\theta}{\partial t^2}\right) \tag{B-16}$$

The last term on the right side of Eqs. B-15 and B-16 is small compared to the other terms, and may be neglected.

We now have differential equations for the three unknown functions, $y(x,t)$, $z(x,t)$ and $\theta(x,t)$. For the static case this system of equations can be reduced to the form

$$\frac{dT}{dx} + \frac{dy}{dx}\frac{d}{dx}\left(T\frac{dy}{dx}\right) + \frac{dz}{dx}\frac{d}{dx}\left(T\frac{dz}{dx}\right) + m_x = 0 \tag{B-17}$$

$$\frac{d^2}{dx^2}\left(EI\frac{d^2 y}{dx^2}\right) + \frac{d^2}{dx^2}\left(T\frac{dz}{dx}\right) - \frac{d}{dx}\left(N\frac{dy}{dx}\right) = p_y \tag{B-18}$$

$$\frac{d^2}{dx^2}\left(EI\frac{d^2 z}{dx^2}\right) - \frac{d^2}{dx^2}\left(T\frac{dy}{dx}\right) - \frac{d}{dx}\left(N\frac{dz}{dx}\right) = p_z \tag{B-19}$$

These equations of statics may be used to gain quantitative knowledge about coupling between bending and torsion. Let us begin with the effect of out of plane bending on torsion. Assuming that $m_x = 0$, one can see that the last two terms in Eq. B-17 represents the bending effect on the torsion. Equation B-17 can be readily transformed into the form,

$$\frac{dT}{dx}\left[1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dz}{dx}\right)^2\right] + \frac{T}{2}\frac{d}{dx}\left[1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dz}{dx}\right)^2\right] = 0 \qquad \text{(B-17a)}$$

Dividing Eq. B-17a by $T\left[1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dz}{dx}\right)^2\right]$, one obtains

$$\frac{1}{T}\frac{dT}{dx} + \frac{1}{2}\frac{1}{1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dz}{dx}\right)^2}\frac{d}{dx}\left[1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dz}{dx}\right)^2\right] = 0 \qquad \text{(B-17b)}$$

Integrating this equation gives

$$T = T_o\left[1 + \left(\frac{dy}{dx}\right)^2 + \left(\frac{dz}{dx}\right)^2\right]^{-1/2} \qquad \text{(B-20)}$$

Thus, for the small angle theory assumed throughout and verified by calculations, the torsion in the beam does not vary with "x" unless there are external moments, m_x, applied continuously or discretely along the length of the riser. Out of plane bending does not create torsion loads or stresses.

APPENDIX C

CABLE DYNAMIC PROGRAM--EFFECTS OF LATERAL GUST LOADING

Equation of Motion

$$\frac{\partial}{\partial S}\left(T_m \frac{\partial \xi}{\partial S}\right) = m_o \frac{\partial^2 \xi}{\partial t^2} - \frac{\rho(x_m)}{2g} DC_D(R_m)P_m^2$$

$$\cdot \left[\left(v_{o_m} + v^t - \frac{\partial \xi}{\partial t}\right)\left| v_{o_m} + v^t - \frac{\partial \xi}{\partial t}\right| - v_{o_m}\left|v_{o_m}\right| \right]$$

where

$$v^t = \left(A \sin \omega t\right) \exp\left\{-\frac{2.3\left(x_m - x_{mid}\right)^2}{1/4\left(x_{A1} - x_{A2}\right)^2}\right\}$$

x_{A1} and x_{A2} bound the forcing function v^t, otherwise $v^t = 0$, and x_{mid} is the cable segment midpoint.

Difference Analogue

$$\frac{1}{4\Delta S^2}\left[T_m\left(\xi_{m+1,n+1} + 2\xi_{m+1,n} + \xi_{m+1,n-1} - \xi_{m,n+1} - 2\xi_{m,n}\right.\right.$$

$$\left. - \xi_{m,n-1}\right) - T_{m-1}\left(\xi_{m,n+1} + 2\xi_{m,n} + \xi_{m,n-1} - \xi_{m-1,n+1}\right.$$

$$\left.\left. - 2\xi_{m-1,n} - \xi_{m-1,n-1}\right)\right]$$

$$= \frac{m_o}{\Delta t^2}\left(\xi_{m,n+1} - 2\xi_{m,n} + \xi_{m,n-1}\right) - \rho(x_m) DC_D(R_m)P_m^2 \left\{\left[v_{o_m} + v^t\right.\right.$$

$$\left. - \left(\frac{\xi_{m,n+1} - \xi_{m,n-1}}{2\Delta t}\right)\right]\left|\left[v_{o_m} + v^t - \left(\frac{\xi_{m,n} - \xi_{m,n-1}}{\Delta t}\right)\right]\right|$$

$$\left. - v_{o_m}\left|v_{o_m}\right|\right\}$$

Boundary Conditions

$$\xi_{1,n+1} = 0$$

$$\xi_{M,n+1} = 0, \quad \text{where } m = 1, 2, \ldots, M$$

Initial Conditions

$$\xi_{m,0} = 0$$

where $m = 1, 2, \ldots, M$

Method of Solution

If $A_{m-1}\xi_{m-1,n+1} + B_{m-1}\xi_{m,n+1} + C_{m-1,n+1}\xi_{m+1,n+1} = D_{m-1}$ for $m = 2, \ldots, M-1$, then

$$A_{m-1} = -\frac{\Delta t^2}{4\Delta S^2 m_o} T_{m-1}$$

$$B_{m-1} = 1 + \frac{\Delta t^2}{4\Delta S^2 m_o}(T_m + T_{m-1}) + \frac{\Delta t\,\rho(x_m) D C_D(R_m) P_m^2}{4 g m_o} \cdot \left| v_{o_m} + v^t - \left(\frac{\xi_{m,n} - \xi_{m,n-1}}{\Delta t}\right)\right|$$

$$C_{m-1} = -\frac{\Delta t^2}{4\Delta S^2 m_o} T_m$$

$$\begin{aligned} D_{m-1} = {} & 2\xi_{m,n} - \xi_{m,n-1} - C_{m-1}\left(2\xi_{m+1,n} + \xi_{m+1,n-1} - 2\xi_{m,n} - \xi_{m,n-1}\right) \\ & + A_{m-1}\left(2\xi_{m,n} + \xi_{m,n-1} - 2\xi_{m-1,n} - \xi_{m-1,n-1}\right) \\ & + \frac{\Delta t\,\rho(x_m) D C_D(R_m) P_m^2}{4 g m_o}\left| v_{o_m} + v^t - \left(\frac{\xi_{m,n} - \xi_{m,n-1}}{\Delta t}\right)\right| \xi_{m,n-1} \\ & - \frac{\Delta t^2 \rho(x_m) D C_D(R_m) P_m^2}{2 g m_o}\left[v_{o_m}\left|v_{o_m}\right| - \left(v_{o_m} + v^t\right)\left| v_{o_m} + v^t \right.\right. \\ & \left.\left. + \left(\frac{\xi_{m,n} - \xi_{m,n-1}}{\Delta t}\right)\right|\right] \end{aligned}$$

The coefficients A_m, B_m, C_m and D_m define a set of M-2 linear equations for the unknowns $\xi_{m,n}$* (m = 2..., M -1). These coefficients are expressed in terms of the previous displacements $\xi_{m,n-1}$, $\xi_{m,n-2}$ which are assumed to be known. The matrix of the linear system for the $\xi_{m,n}$ takes the form of a three term M-2 by M-2 band diagonal matrix. This matrix is triangularized by use of gaussian elimination techniques and is readily inverted, thus yielding the solution for the $\xi_{m,n}$. The solution for the $\xi_{m,n}$ is thus carried out point-by-point in time, beginning with the initial conditions for $\xi_{m,n}$, i.e., $\xi_{m,o} = 0$. In other words at each time $n\Delta t$, the complete set $\xi_{m,n}$ (m = 2..., M -1) is obtained and then together with previous $\xi_{m,n-1}$ etc., is used to solve for the complete set $\xi_{m,n+1}$.

If the following computations are made initially,

$$C_1 = \frac{C_1}{B_1}$$

$$D_1 = \frac{D_1}{B_1}$$

The generating sequences may be given for m = 1,2,...,M-3 as

$$B_{m+1} = B_{m+1} - A_{m+1}C_m$$

$$C_{m+1} = \frac{C_{m+1}}{B_{m+1}}$$

$$D_{m+1} = \frac{D_{m+1} - A_{m+1}D_m}{B_{m+1}}$$

*(The subscript m labels the space net; the subscript n labels the time net.)

Following the computation of the above sequences, the solution for $\xi(m,t)$ may be written for $m = M-2, M-3, \ldots, 2$ as

$$\xi_{M-1,n+1} = D_{M-1}$$
$$\xi_{m,n+1} = D_{m-1} - C_{m-1}\xi_{m+1,n+1}$$

Required Data

a) Input constants . m_o, g, D

b) Static results. $x(S)$, $T(S)$, $P(S)$ at intervals ΔS, cable segment endpoints $x_1(S)$ and $x_2(S)$ ($x_1 > x_2$) and endpoints of forcing function range $x_{A1}(S)$ and $x_{A2}(S)$ ($x_{A1} > x_{A2}$)*

c) Supporting subprograms to compute. v_o, ρ, C_D, and ν

*Note: For the remainder of the discussion X will be used in place of x for the altitude coordinate.

Card Input to Program Balloon

Columns	10	20	30	40	50	60	70
Card 1	NDA	NMO	NYR				
Card 2	NTAB	NXPRT	NVEL				
Card 3	IDEBM	IDEBB	IDEBA	IDEBG	IDEBS		
Card 4	X1	X2	EPS	TTEST	DTPRT	DTMPRT	DT
Card 5	DS	G	D	EM0	XBALL	SBALL	OMEGA
Card 6	A	XA1	XA2	XMID			

Columns	12	24	36	48	60	
Card Group 7	XV(1)	V(1)	XV(2)	V(2)	XV(3)	V(3)
	XV(NVEL	V(NVEL)				

Tape Input to Program Balloon

Logical tape 2 contains 3 binary records.

a) Record 1. X(1),...,X(NTAB)

b) Record 2. T(1),...,T(NTAB)

c) Record 3. P(1),..., P(NTAB

Nomenclature

NDA, NMO, NYR date

NTAB number of data points tabulated by static program

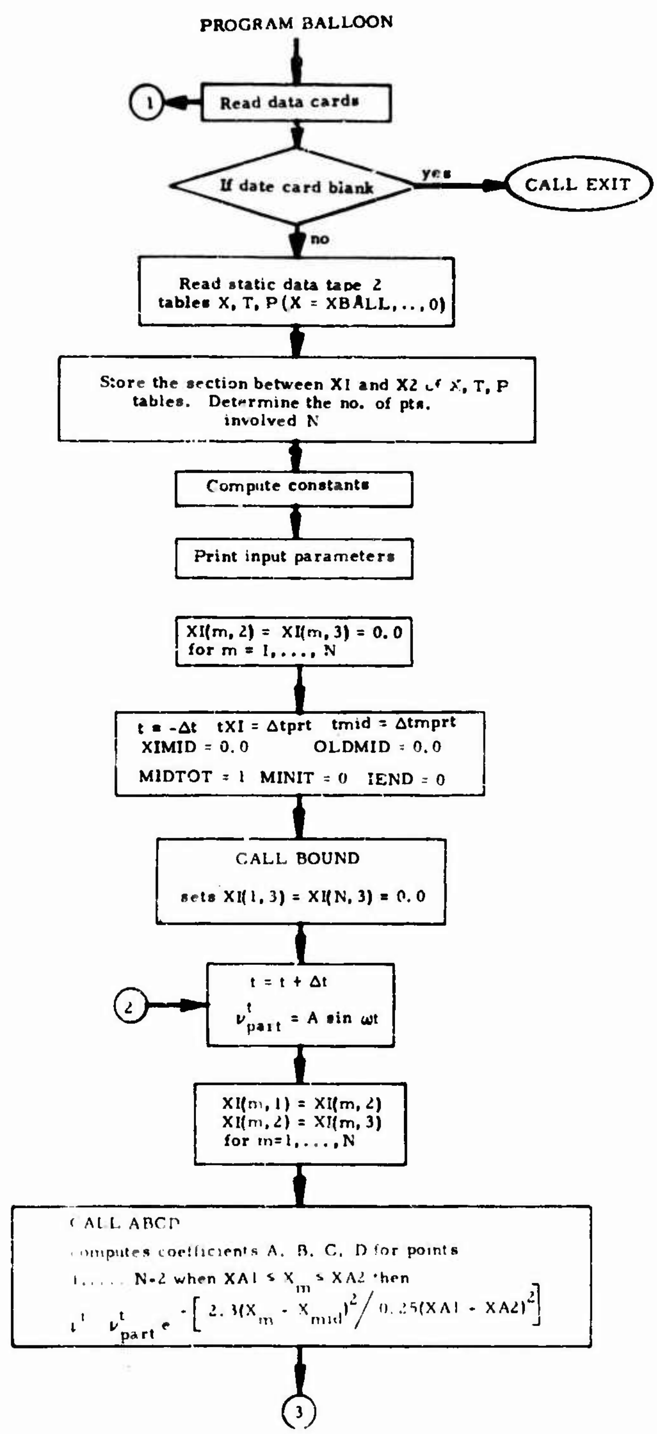
NXPRT increment in X to use in printing the cable section results (i.e., print every NXPRT x)

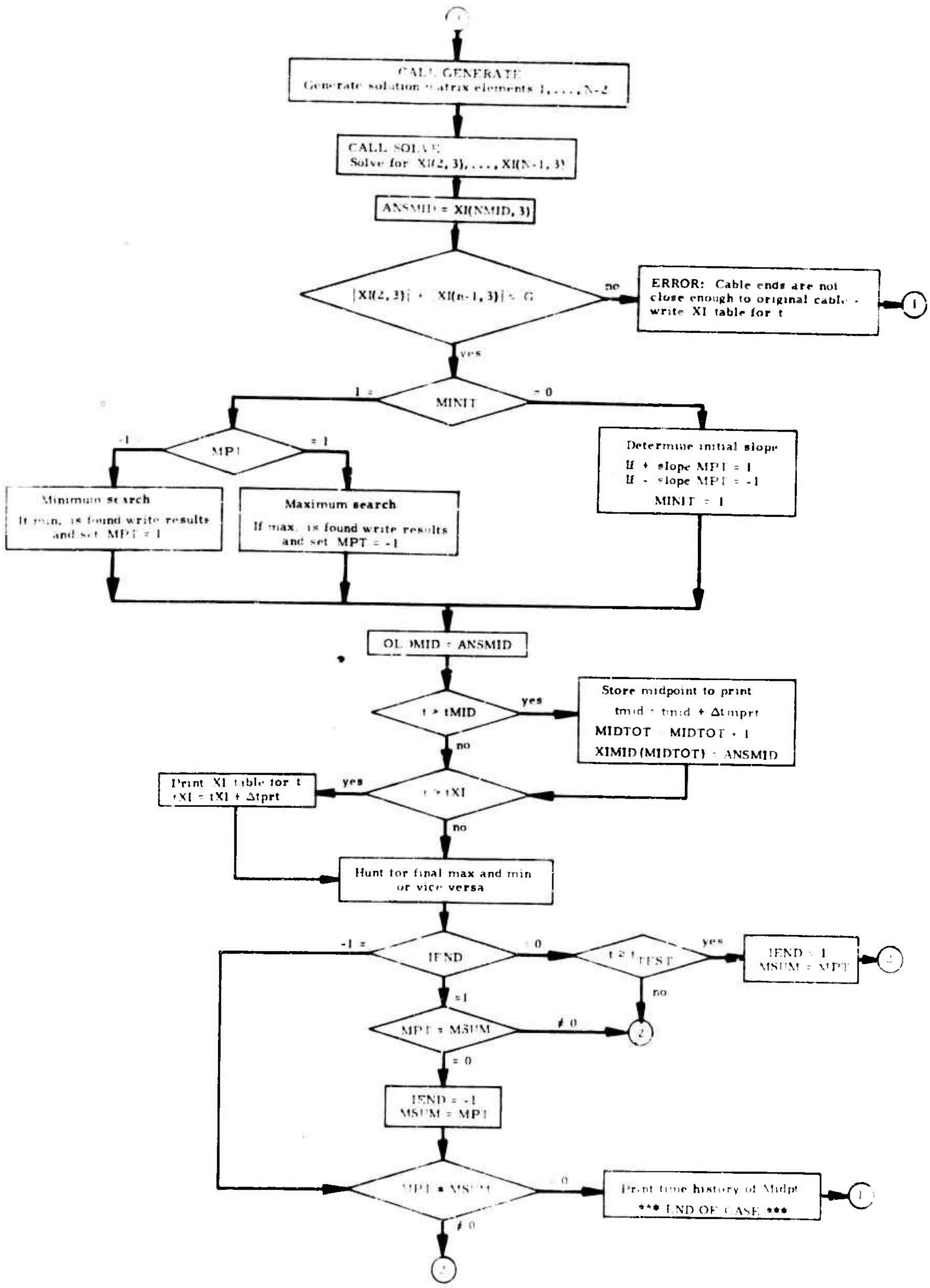
NVEL	number of entries in wind velocity tables
IDEB	card--leave blank--used for debugging
X1	X value at upper end of cable segment, meters
X2	X value at lower end of cable segment X1 > X2, meters
EPS	ϵ = tolerance for deflection at ends of cable segment
TTEST	time to test the program and halt at the first min and max after t = TTEST
DTPRT	increment in t to use in printing the cable section results
DTMPRT	increment in t to use in printing the time history of the cable section midpoint
DT	Δt = increment in t to use in evaluating the differential equations
DS	ΔS = increment in S at which the static date was recorded, meters
G	gravity at sea level, meters/sec^2
D	diameter of cable, meters
EM0	m_o = initial mass per unit length of cable, Kg/meters
XBALL	x-coordinate of balloon, meters
SBALL	length of cable, meters
OMEGA	ω = angular frequency of forcing function, radians
A	amplitude of forcing function v^t, meters/sec
XA1	upper limit of forcing function v^t, meters
XA2	lower limit of forcing function v^t, meters
XMID	cable section midpoint, meters

XV altitude in wind velocity table, meters

V velocity in wind velocity table, meters/sec

Flow Sheet (following pages)


PROGRAM BALLOON
1
Read data cards
If date card blank
yes
CALL EXIT
no
Read static data tape 2
tables X, T, P (X = XBALL, ..., 0)
Store the section between XI and X2 of X, T, P
tables. Determine the no. of pts.
involved N
Compute constants
Print input parameters
XI(m, 2) = XI(m, 3) = 0.0
for m = 1, ..., N
t = -Δt tXI = Δtprt tmid = Δtmprt
XIMID = 0.0 OLDMID = 0.0
MIDTOT = 1 MINIT = 0 IEND = 0
CALL BOUND
sets XI(1, 3) = XI(N, 3) = 0.0
2
t = t + Δt
ν^t_{part} = A sin ωt
XI(m, 1) = XI(m, 2)
XI(m, 2) = XI(m, 3)
for m=1, ..., N
CALL ABCD
computes coefficients A, B, C, D for points
1, ..., N-2 when XA1 ≤ X_m ≤ XA2 then
$\nu^t = \nu^t_{part} e^{-\left[2.3(X_m - X_{mid})^2 / 0.25(XA1 - XA2)^2\right]}$
3


CALL GENERATE
Generate solution matrix elements 1, . . . , N-2
CALL SOLVE
Solve for XI(2, 3), . . . , XI(N-1, 3)
ANSMID = XI(NMID, 3)
|XI(2, 3)| + |XI(n-1, 3)| ≤ G
no
ERROR: Cable ends are not close enough to original cable - write XI table for t
yes
MINIT
Determine initial slope
If + slope MPT = 1
If - slope MPT = -1
MINIT = 1
MPT
Minimum search
If min. is found write results and set MPT = 1
Maximum search
If max. is found write results and set MPT = -1
OLDMID = ANSMID
t > tMID
Store midpoint to print
tmid = tmid + Δtmprt
MIDTOT = MIDTOT + 1
XIMID (MIDTOT) = ANSMID
Print XI table for t
tXI = tXI + Δtprt
t > tXI
Hunt for final max and min or vice versa
IEND
IEND = 1
MSUM = MPT
MPT + MSUM
IEND = -1
MSUM = MPT
Print time history of Midpt
*** END OF CASE ***

```
      PROGRAM BALLOON
C     DYNAMIC ANALYSIS OF A 2-D TETHERED BALLOON SYSTEM    SN344-712
      DIMENSION  SPACE(10000),XI(2000,3),TTAB(2000),PTAB(2000)
     1,XTAB(2000)
      DIMENSION XV(100),V(100),XIMID(1000),STUFP(10000),STUFT(10000)
      COMMON SPACE,XI,TTAB,PTAB,IDEBM,IDEBB,IDEBA,IDEBG,IDEBS
     1,XTAB
      COMMON /1/ STUFP,STUFT
C     N POINTS DESCRIBE INPUT CABLE SECTION BETWEEN X1 AND X2
C     WHERE X1 MORE THAN X2.
C     CABLE ANALYZED FROM X1 TO X2 AT EACH DT
C     THE CABLE IS SBALL LONG AND HAS NTAB INPUT INFORMATION POINTS
C     AT EVERY DS POINT AND IS RECORDED ON TAPE UNIT 2
C     NVEL IS NO. OF PTS. IN VELOCITY TABLE OF ALTITUDE XV VS VELOCITY V
C     IDEB INDICES PRINT INTERMEDIATE OUTPUT IF NOT EQUAL TO ZERO
C     DISPLACEMENT IS PRINTED AT EVERY NXPRT X VALUE IN TIME INCREMENTS
C     OF DTPRT
C     DISPLACEMENTS AT THE MID POINT OF THE CABLE WILL BE PRINTED AT
C     TIME INCREMENT DTMPRT
C     WHEN T=TTEST THE PROGRAM IS STOPPED AT THE FIRST MAX AND MIN
C     AFTER TTEST
C     1ST DISPLACEMENT AFTER S1 AND LAST BEFORE S2 MUST BE WITHIN EPS
C     V TO THE T =  A .SIN(OMEGA.T) . EXPONENTIAL FUNCTION OF X
C     SPACE(1-2000)=A, (2001-4000)=B, (4001-6000)=C, (6001-8000)=D
C     SPACE(8001-10000)= D.RHO(XM).CD(RM)P(XM)/(2G)
    1 READ (5,8000) NDA,NMO,NYR
      IF (NDA) 99,99,2
    2 READ (5,8000)   NTAB,NXPRT,NVEL
      READ (5,8000)IDEBM,IDEBB,IDEBA,IDEBG,IDEBS
 8000 FORMAT(7I10)
      READ (5,8001)X1,X2,EPS,TTEST,DTPRT,DTMPRT,DT,DS, G,D,EMO,XBALL,
     1SBALL,OMEGA
 8001 FORMAT (7F10.8)
      READ (5,8001) A,XA1,XA2 ,XMID
 8002 FORMAT(6E12.6)
      READ (5,8002) (XV(I),V(I),I=1,NVEL)
C     INPUT X,T,P
      REWIND 2
      WRITE (6,9050)NTAB
 9050 FORMAT (//,53X,7H NTAB =,I6,//)
      READ (2)(SPACE(I),I=1,NTAB)
      READ (2)(STUFT(I),I=1,NTAB)
      READ (2)(STUFP(I),I=1,NTAB)
      DO 30 I=1,NTAB
      INDEX=I
      IF (X1-SPACE(I))30,40,40
   30 CONTINUE
   40 DO 50 I=INDEX,NTAB
      NXA1=I
      IF (XA1-SPACE(I)) 50,60,60
   50 CONTINUE
   60 DO 62 I=NXA1,NTAB
      NMID=I
      IF (XMID-SPACE(I)) 62,65,65
   62 CONTINUE
   65 DO 70 I=NMID,NTAB
      NXA2=I
      IF (XA2-SPACE(I)) 70,80,80
   70 CONTINUE
   80 DO 90 I=NXA2,NTAB
      LAST=I
```

```
      IF (XA2 -SPACE(I)) 90,95,95
   90 CONTINUE
   95 N=LAST-INDEX+1
      NXA1=NXA1-INDEX+1
      NXA2=NXA2-INDEX+1
      NMID=NMID-INDEX+1
      WRITE (6,7000) NTAB,N,INDEX,LAST,NXA1,NXA2,XTAB(1),TTAB(1),PTAB(1)
     1, XTAB(N),TTAB(N),PTAB(N)
 7000 FORMAT(6I10,/,6E20.8)
      WRITE(6,9000)NDA,NMO,NYR
 9000 FORMAT(1H1,30X,45HDYNAMIC ANALYSIS OF A TETHERED BALL-SN344-712,
     15X,I2,1H/,I2,1H/,I2,//)
      K=1
      DO 100 I=INDEX,LAST
      XTAB(K)=SPACE(I)
  100 K=K+1
      WRITE (6,9001)N,X1,X2,XBALL
 9001 FORMAT(5X,17H CABLE DEFINED BY,I5,15H POINTS FROM X=,E15.8,
     1 6H TO X=,E15.8,20H WHERE X AT BALLOON=,E15.8)
      K=1
      DO 110 I=INDEX,LAST
      TTAB(K)=STUFT(I)
  110 K=K+1
      WRITE (6,9002) DT,TTEST,DTPRT,DTMPRT,EPS
 9002 FORMAT (19H TIME INCREMENT DT=,E12.5,4X,13HTIME TO TEST=,E12.5,4X,
     115HDT TO PRINT XI=,E12.5,4X,24HDT TO PRINT XI AT MIDPT=,E12.5,/,43
     2X,17H TOLERANCE ON XI=, E15.8,/)
      K=1
      DO 120 I=INDEX,LAST
      PTAB(K)=STUFP(I)
  120 K=K+1
      WRITE (6,9003) G,D,EMO,DS,SBALL,OMEGA
 9003 FORMAT(9X,2H G,18X,2H D,18X,2HMO,18X,2HDS,13X,12HS OF BALLOON,11X,
     15HOMEGA,/,E17.8,5E20.8)
C     SET UP CONSTANTS
      C1=DT*DT/EMO
      C2=0.5*C1/(DS*DS)
      C3=0.5*DT/EMO
      DXX=0.5*(XA1-XA2)
      ALPHA=2.3025851/(DXX*DXX)
      J=8001
      NS=1
      DO 150 I=1,N
      X=XTAB(I)
      CALL RHOX(X,RHO)
      CALL NU (X,GNU)
      CALL INTER(NVEL,NS,X,V,XV,VEL)
      R=ABS(VEL*D*PTAB(I)/GNU)
      CALL CDR(R,CD)
      SPACE(J)=0.5*RHO*D*CD*PTAB(I)*PTAB(I)/G
      PTAB(I)=VEL
  150 J=J+1
C     VELOCITY NOW STORED IN PTAB
      WRITE (6,9004)A,XA1,XA2
 9004 FORMAT( 7X,41H V TO THE T=A.SIN(OMEGA.T).F(X) WHERE A =,E15.8,12H
     1BETWEEN X =,E15.8,8H AND X =,E15.8)
      COUNT=INDEX+NMID -2
      SMID=SBALL-DS*COUNT
      TMIDPT=TTAB(NMID)
C     PRINT INPUT
      IF(IDEBM) 10,20,10
```

```
   10 WRITE (6,9005)
      WRITE (6,9006)(XV(I),V(I),I=1,NVEL)
 9005 FORMAT(26X,28H1VELOCITY TABLE     ALTITUDE,12X,8HVELOCITY,/)
 9006 FORMAT(37X,2E20.8)
   20 CONTINUE
C     INITIALIZE
      T=-DT
      TXI=DTPRT
      TMID=DTMPRT
      TSTOP=2.0*TTEST
C     INITIAL CONDITIONS
      DO 200 I=1,N
      XI(I,2)=0.0
  200 XI(I,3)=0.0
      XIMID(1)=0.0
      MIDTOT=1
      MINIT=0
C     THERE HAVE BEEN MIDTOT MIDPTS STORED. MINIT=0 MEANS INITIAL TIME
      OLDMID=0.0
      IEND=0
      VALS1=0.0
      VALS2=0.0
      CALL BOUND(N,VALS1,VALS2)
C     SET BOUNDARY CONDITIONS--XI=0,AT ENDS OF CABLE SEGMENT
C     RESET LOOP
  300 T=T+DT
      IF (T-TSTOP) 301,800,800
  301 SVT=A *SIN(OMEGA*T)
      DO 250 I=1,N
      XI(I,1)=XI(I,2)
  250 XI(I,2)=XI(I,3)
C     COMPUTE COEFFICIENTS A,B,C,D OF XI AT N+1 SUCH THAT
C     A(M-1) XI(M-1)  +  B(M-1) XI(M)  +  C(M-1) XI(M+1)  =  D(M-1)
      CALL ABCD(N,C1,C2,C3,SVT,NXA1,NXA2,DT,ALPHA,XMID)
C     GENERATE ELEMENTS IN THE SOLUTION MATRIX
      CALL GENERAT (N)
C     SOLVE THE TRIANGULAR SOLUTION MATRIX
      CALL SOLVE(N)
      ANSMID=XI(NMID,3)
C     CHECK THAT DISPLACEMENT IS SMALL NEAR THE ENDS
      NM=N-1
      IF ((ABS(XI(2,3))+ABS(XI(NM,3)))-EPS) 350,350,340
  340 WRITE (6,9010) T,XI(2,3),XI(NM,3),EPS
 9010 FORMAT(6H1AT T=,E13.6,18H XI AT THE 2ND PT=,E13.6,22H AND XI AT TH
     1E N-1 PT=,  E13.6,21H WHERE THE TOLERANCE=,E13.6)
      WRITE (6,9022) T
      WRITE (6,9023)(XTAB(I),TTAB(I),XI(I,3),I=1,N,NXPRT)
      GO TO 1
  350 CONTINUE
C     FIND MAX,MIN OF TIME HISTORY OF CABLE MIDPOINT
      IF (MINIT) 410,380,410
  380 MINIT=1
      IF (ANSMID-OLDMID)  400,390,390
C     POSITIVE SLOPE FIND MAX
  390 MPT=1
      GO TO 500
C     NEGATIVE SLOPE  FIND MIN
  400 MPT=-1
      GO TO 500
  410 IF(MPT) 420,450,450
C     MINIMUM SEARCH
```

```
  420 IF (ANSMID-OLDMIN) 500,440,440
C       MINIMUM
  440 MPT=1
      WRITE (6,9020) ANSMID,T,SMID,XMID
 9021 FORMAT(/,37H MAXIMUM DEFLECTION AT MIDPOINT---XI=,E15.8,4X,
     15HTIME=,E15.8,4X,2HS=E15.8,4X,2HX=,E15.8,/)
      GO TO 500
C       MAXIMUM SEARCH
  450 IF(ANSMID-OLDMID) 460,460,500
C       MAXIMUM
  460 MPT=-1
      WRITE (6,9021)ANSMID,T,SMID,XMID
 9020 FORMAT(/,37H MINIMUM DEFLECTION AT MIDPOINT---XI=,E15.8,4X,
     15HTIME=,E15.8,4X,2HS=E15.8,4X,2HX=,E15.8,/)
  500 OLDMIN=ANSMID
      IF(T-TMID) 600,510,510
  510 TMID=TMID+DTMPRT
      MIDTOT=MIDTOT+1
      XIMID(MIDTOT)=ANSMID
  600 IF (ABS(T-TXI)-0.01) 610,610,700
  610 TXI=TXI+DTPRT
      WRITE (6,9022) T
 9022 FORMAT(//,11X,6H TIME=E15.8,7X,2H X,18X,2H T,18X,2HXI)
      WRITE (6,9023)(XTAB(I),TTAB(I),XI(I,3),I=1,N,NXPRT)
 9023 FORMAT(27X,3E20.8)
  700 IF (IEND) 770,710,750
  710 IF(T-TTEST) 300,720,720
  720 MSUM=MPT
      IEND=1
      GO TO 300
  750 IF(MPT+MSUM) 300,760,300
  760 IEND=-1
      WRITE (6,9022) T
      WRITE (6,9023)(XTAB(I),TTAB(I),XI(I,3),I=1,N,NXPRT)
      MSUM=MPT
  770 IF (MPT+MSUM) 300,800,300
  800 WRITE (6,9022) T
      WRITE (6,9023)(XTAB(I),TTAB(I),XI(I,3),I=1,N,NXPRT)
      WRITE (6,9025) SMID,XMID,TMIDPT
 9025 FORMAT(1H1,13X,39HTIME HISTORY OF CABLE MIDPOINT WHERE S=,E15.8,
     13X,2HX=,E15.8,3X,2HT=,E15.8,/,49X,2HT ,18X,2HXI,/)
      TIME=0.0
      DO 900 I=1,MIDTOT
      WRITE (6,9026) TIME,XIMID(I)
 9026 FORMAT(37X,2E20.8)
  900 TIME=TIME+DTMPRT
      WRITE (6,9030)
 9030 FORMAT(//,49X,22H *****END OF CASE*****)
      GO TO 1
   99 CALL EXIT
      END
      SUBROUTINE ABCD(N, C1,C2,C3,SVT,NXA1,NXA2,DT,ALPHA,XMID)
      DIMENSION  SPACE(10000),XI(2000,3),TTAB(2000),PTAB(2000)
     1,XTAB(2000)
      COMMON SPACE,XI,TTAB,PTAB,IDEBM,IDEBB,IDEBA,IDEBG,IDEBS
     1,XTAB
C       COMPUTES N-2 COEFICIENTS A,B,C,AND D DEFINING XI AT N+1
C       GIVEN XI AT N AND AT N-1
C       SPACE(1-2000)=A, (2001-4000)=B, (4001-6000)=C, (6001-8000)=D
C       XI(M,1)=XI(M,N-1),XI(M,2)=XI(M,N)
      NM =N- 1
```

```
      DO 100 M=2,NM
      MM=M-1
      MP=M+1
      SLOP=SPACE(6000+M)
C     ADJUST FORCING FUNCTION
      VEL=PTAB(M)
      VV=VEL*ABS(VEL)
      IF (M-NXA1) 60,50,40
   40 IF( M-NXA2) 50,50,60
   50 DXX=XTAB(M)-XMID
      POWER=-ALPHA*DXX*DXX
C     SVT=A.SIN(OMEGA.T)
      VT=SVT*EXP(POWER)
      CON=VEL+VT
      GO TO 70
   60 CON=VEL
   70 CONABS=ABS(CON-(XI(M,2)-XI(M,1))/DT)
      CONST=C3*SLOP*CONABS
      A=-TTAB(MM)*C2
      B=1.0+(TTAB(M)+TTAB(MM))*C2+CONST
      C=-TTAB(M)*C2
      D=-XI(M,1)+2.*XI(M,2)-C*(2.0*XI(MP,2)+XI(MP,1)-2.0*XI(M,2)
     1-XI(M,1))+A*(2.0*XI(M,2)+XI(M,1)-2.0*XI(MM,2)-XI(MM,1))
     2 +CONST*XI(M,1) -C1*SLOP*(VV-CON*CONABS)
      SPACE(MM)=A
      SPACE(MM+2000)=B
      SPACE(MM+4000)=C
  100 SPACE(MM+6000)=D
      RETURN
      END
      SUBROUTINE  GENERAT (N)
      DIMENSION  SPACE(10000),XI(2000,3),TTAB(2000),PTAB(2000)
     1,XTAB(2000)
      COMMON SPACE,XI,TTAB,PTAB,IDEBM,IDEBB,IDEBA,IDEBG,IDEBS
     1,XTAB
C     GENERATE ELEMENTS IN THE SOLUTION MATRIX
C     SPACE(1-2000)=A, (2001-4000)=B, (4001-6000)=C, (6001-8000)=D
C     GENERATES C(1)-C(N-1),D(1)-D(N-1)
      SPACE(4001)=SPACE(4001)/SPACE(2001)
      SPACE(6001)=SPACE(6001)/SPACE(2001)
      NM=N-3
      DO 100 M=1,NM
      MP=M+1
      M2000=MP+2000
      M4000=MP+4000
      M6000=MP+6000
      SPACE(M2000)=SPACE(M2000)-SPACE(MP)*SPACE(M+4000)
      SPACE(M4000)=SPACE(M4000)/SPACE(M2000)
  100 SPACE(M6000)=(SPACE(M6000)-SPACE(MP)*SPACE(M+6000))/SPACE(M2000)
      RETURN
      END
      SUBROUTINE SOLVE (N)
      DIMENSION  SPACE(10000),XI(2000,3),TTAB(2000),PTAB(2000)
     1,XTAB(2000)
      COMMON SPACE,XI,TTAB,PTAB,IDEBM,IDEBB,IDEBA,IDEBG,IDEBS
     1,XTAB
C     SOLVE THE TRIANGULAR SOLUTION MATRIX
C     SPACE(1-2000)=A, (2001-4000)=B, (4001-6000)=C, (6001-8000)=D
      NM=N-2
      INDEX=NM+6000
      XI(N- 1,3)=SPACE(INDEX)
```

```
      DO 100 I=2,NM
      J=NM -I+1
  100 XI(J+1,3)=SPACE(J+6000)-SPACE(J+4000)*XI(J+2,3)
      RETURN
      END
      SUBROUTINE  BOUND (N,VALS1,VALS2)
      DIMENSION  SPACE(10000),XI(2000,3),TTAB(2000),PTAB(2000)
     1,XTAB(2000)
      COMMON SPACE,XI,TTAB,PTAB,IDEBM,IDEBB,IDEBA,IDEBG,IDEBS
     1,XTAB
C     SET BOUNDARY CONDITIONS--XI=0,DXI/DT=0 AT ENDS OF CABLE SEGMENT
      XI(1,3)=VALS1
      XI(N,3)=VALS2
      RETURN
      END
      SUBROUTINE NU (X,GNU)
C     KINEMATIC VISCOSITY VS ALTITUDE
      IF (X-10000.) 1,1,2
    1 GNU=-4.823+X/26800.
      GO TO 5
    2 IF (X-17700.) 3,3,4
    3 GNU=X/17100.-5.035
      GO TO 5
    4 GNU=X/14250.-5.2421
    5 GNU=10.**GNU
      RETURN
      END
      SUBROUTINE INTER (N,NS,XL,V,XV,SVL)
C     GIVEN THE ALTITUDE, THIS ROUTINE INTERPOLATES LINEARLY TO
C     OBTAIN THE WIND VELOCITY FROM INPUT TABLE
      DIMENSION  V(100),XV(100)
      NN=N-NS+1
      IF (XL-XV(NN)) 5,5,6
    6 NS=1
    5 DO 1 I=NS,N
      J=N-I+1
      IF (XL-XV(J)) 1,3,4
    3 SVL=V(J)
      GO TO 2
    4 SVL=(V(J+1)-V(J))*(XL-XV(J+1))/(XV(J+1)-XV(J))+V(J+1)
      GO TO 2
    1 CONTINUE
    2 NS=J+1
      RETURN
      END
      SUBROUTINE CDR(R,CD)
C     DRAG COEFFICIENT VS REYNOLDS NUMBER FOR CABLE
      IF (R-2.23) 1,1,2
    1 CD=10.8*R**(-.742)
      GO TO 10
    2 IF (R-8.0) 3,3,4
    3 CD=9.15*R**(-.526)
      GO TO 10
    4 IF (R-1000.0) 5,5,6
    5 CD=4.95*R**(-.232)
      GO TO 10
    6 IF (R-10000.) 7,7,8
    7 CD=1.0
      GO TO 10
    8 CD=1.15
   10 RETURN
```

```
      END
      SUBROUTINE RHOX(X,RHO)
C     MASS DENSITY VS ALTITUDE
      RHO=.25-X/16000.
      RHO=10.**RHO
      RETURN
      END
```

APPENDIX D

WIND INDUCED VIBRATIONS BY MEANS OF VORTEX SHEDDINGS

Introduction

Considering the cable subjected to a steady wind flow, any high frequency dynamic effect on the cable will cause it to vibrate about its static configuration. In other words, the dynamic response of the cable to high frequency excitations can be considered as a perturbation on a static initial configuration which is related to a steady wind. Further simplification of the problem can be done by considering the high tension in the cable and its low weight per unit length. The propagation velocity of the transverse wave in the cable is

$$C = \sqrt{\frac{T}{m}} \tag{D-1}$$

In addition the frequency of shedding vortex pairs is

$$f_v = \frac{S_T(R)V_o P}{D} \tag{D-2}$$

where S_T is the well known Strouhal number. In the present case f_v is of the order of 1000 cps. The wavelength which one should be concerned with is

$$2L = \frac{C}{f_v}$$

which, for the present purposes, is of the order of 2 meters. Hence the most effective mode of the cable will be the one which is of wavelength 2L.

In order to estimate the response of the cable to vortex sheddings, one can solve the equation of motion for the cable considering a portion which has the length L. This portion can be assumed to be supported at its endpoints.

The perturbed dynamic equation of motion for the cable is

$$T \frac{\partial^2 W}{\partial S^2} = m \frac{\partial^2 W}{\partial t^2} + \frac{1}{2} \rho C_D D \sqrt{V_o^2 P^2 + \left(\frac{\partial W}{\partial t}\right)^2} \frac{\partial W}{\partial t}$$

$$- \frac{1}{2} \rho C_K V_o^2 P^2 D \sin 2\pi f_v t \qquad \text{(D-3)}$$

where C_K is the lift coefficient (Refs. 5 and 6) and $V_o P$ is the normal component of the wind velocity vector with respect to the cable.

The last equation, which has been solved numerically by means of finite differences is discussed in a following section. The scheme that was used for this purpose was an unconditionally stable one. The integration process ran on a computer for two hundred cycles of the forcing function. Within this process of integration the cable reached a steady state response. For $L = 0.775$ meters the maximum deflection of the cable was 0.58×10^{-4} meter, which does not indicate any considerable increase in the stresses in the cable.* In conclusion, the cable subjected to high tension converts, by means of the vortex sheddings, wind flow energy to other kinds of energy; however, there is no indication that the high frequency vibrations of the cable with small amplitude might cause any failure.

*The properties of the cable and wind field were chosen from the steady-state example and are as follows: midpoint of cable portion at 4000 m; $T = 773.0$ Kgf, $P = 0.522$, $\rho = 1.0$ Kg/m^3, $V_o = 13.8$ m/sec, $m = 8 \times 10^{-4}$ Kg/m, $D = 2.5 \times 10^{-3}$ m, $C_D = 1$, $C_K = 0.76$, $S_T = 0.22$.

Numerical Analysis of Problem

a) Equation of Motion

$$\frac{\partial}{\partial S}\left(T_m \frac{\partial W}{\partial S^2}\right) = m_o \frac{\partial^2 W}{\partial t^2} - \frac{\rho\left(x_m\right)}{2g} D V_o^2 P_m^2 V^t C_K$$

$$+ \frac{\rho\left(x_m\right)}{2g} C_D \left(R_m\right) D$$

$$\cdot\left[V_o^2 P_m^2 + \left(\frac{\partial W}{\partial t}\right)^2\right]^{1/2} \frac{\partial W}{\partial t}$$

where W is a deflection $\perp$ to the plane of Fig. 1. The short cable section length leads to assuming the following parameters constant: V_o, T_m, P_m, $\rho(x_m)$, $C_D\left(R_m\right)$, $C_K\left(R_m\right)$, $S_T\left(R_m\right)$

The equation of motion becomes:

$$T \frac{\partial^2 W}{\partial S^2} = m_o \frac{\partial^2 W}{\partial t^2} - \frac{\rho}{2g} C_K D V_o^2 P^2 V^t + \frac{\rho}{2g} C_D D$$

$$\cdot\left[V_o^2 P^2 + \left(\frac{\partial W}{\partial t}\right)^2\right]^{1/2} \frac{\partial W}{\partial t}$$

where:

$$V^t = \sin 2\pi f_v t$$

$$f_v = \frac{S_T V_o P}{D}$$

given the frequency f_v, the following parameters can be calculated:

$$\Delta t = \frac{1}{20 f_v} \qquad \ell = \frac{\sqrt{\frac{T}{m_o}}}{2f_v} \qquad t_{max} = \frac{2\pi}{f_v} \cdot (\text{no. cycles})$$

b) Difference Analogue

$$\frac{T}{4\Delta S^2}\left\{W_{m+1,n+1} + W_{m-1,n+1} - 2W_{m,n+1} + 2\left(W_{m+1,n}\right.\right.$$

$$\left.\left. + W_{m-1,n} - 2W_{m,n}\right) + W_{m+1,n-1} + W_{m-1,n-1} - 2W_{m,n-1}\right\}$$

$$= \frac{m_o}{\Delta t^2}\left(W_{m,n+1} - 2W_{m,n} + W_{m,n-1}\right) - \frac{\rho}{2g} C_K D V_o^2 P^2 V^t$$

$$+ \frac{\rho C_D D}{2g}\left[V_o^2 P^2 + \frac{\left(W_{m,n} - W_{m,n-1}\right)^2}{\Delta t^2}\right]^{1/2}\left(\frac{W_{m,n+1} - W_{m,n-1}}{2\Delta t}\right)$$

The cable segment is analyzed for half of its length ℓ and is defined between points 1 and M where M is at $\ell/2$. The "ficticious" point M + 1 is used.

c) Initial Conditions

$W_{m,o} = 0.0$ for $m = 1, 2, \ldots, M+1$

d) Method of Solution

If $A_{m-1} W_{m-1,n+1} + B_{m-1} W_{m,n+1} + C_{m-1}, W_{m+1,n+1}$

$= D_{m-1}$ for $m = 2. \ldots, M$

then:

$$A_{m-1} = - \frac{\Delta t^2 T}{4\Delta S^2 m_o}$$

$$B_{m-1} = 1 - 2A_{m-1} + \frac{\Delta t \rho C_D D}{4m_o g}\left[V_o^2 P^2 + \frac{\left(W_{m,n} - W_{m,n-1}\right)^2}{\Delta t^2}\right]^{1/2}$$

$$C_{m-1} = A_{m-1}$$

$$D_{m-1} = - A_{m-1}\left[W_{m+1,n-1} + W_{m-1,n-1} - 2W_{m,n-1} + 2\left(W_{m+1,n} + W_{m-1,n} - 2W_{m,n}\right)\right] + \frac{\Delta t^2 \rho C_K D V_o^2 P^2 V^t}{2m_o g} + \frac{\Delta t \rho C_D D}{4m_o g}\left[V_o^2 P^2 + \frac{\left(W_{m,n} - W_{m,n-1}\right)^2}{\Delta t^2}\right]^{1/2} W_{m,n-1} + 2W_{m,n} - W_{m,n-1}$$

The solution of the linear system for $W_{m,n}$ is carried out by the matrix inversion procedure described in Appendix C.

e) Boundary Conditions

$$W_{1,n+1} = 0.0$$

$$A_{M-1} = A_{M-1} + C_{M-1}$$

$$D_{M-1} = D_{M-1} + C_{M-1}\left[W_{M+1,n-1} - W_{M-1,n-1} + 2\left(W_{M+1,n} - W_{M-1,n}\right)\right]$$

f) Solution of System of Equations

If the following computations are made initially,

$$C1 = C1/B1 \qquad D1 = D1/B1$$

The generating sequences may be given for $m = 1, \ldots, M - 1$

$$B_{m+1} = B_{m+1} - A_{m+1} C_m$$

$$C_{m+1} = \frac{C_{m+1}}{B_{m+1}}$$

$$D_{m+1} = \frac{D_{m+1} - A_{m+1} D_m}{B_{m+1}}$$

Following the computations of the above sequences the solution for $W(m, t)$ may be written for $m = M+1, M, \ldots\ldots, 2$

$$W_{M,n+1} = D_{M-1}$$

$$W_{M+1,n+1} = W_{M-1,n+1} - 2\left(W_{M+1,n} - W_{M-1,n}\right) - W_{M+1,n-1} + W_{M-1,n-1}$$

$$W_{m,n+1} = D_{m-1} - C_{m-1} W_{m+1,n+1}$$

where $m = M-1, M-2, \ldots, 2$

g) Required Data

Input constants: D, m_o, g, V_o, T, P, ρ, C_D, C_K, S_T

where C_D, C_K, and S_T are functions of Reynolds No. R

Input to Program MUSIC

Columns	10	20	30	40	50	60	70
Card 1	NDA	NMO	NYR				
Card 2	NPOINT	NCYCLE	NXPRT	NTPRT	IDEBUG		
Card 3	G	X	DS	T	P	RHO	V0
Card 4	EM0	D	CD	CK	ST		

Notation

NDA, NMO, NYR — Date

NPOINT — Number of points in the cable section to be analyzed. The section will be analyzed up to its midpoint M = (NPOINT/2) + 1

NCYCLE — Number of cycles to run program

NXPRT — Increment used in printing deflections W at any time (i.e., printed at every NXPRT point out of M + 1 points)

NTPRT — Increment used in printing the time history of the midpoint (i.e., printed at every NTPRT time)

IDEBUG — If IDEBUG = 1 deflections at every NTPRT time will be printed
If IDEBUG = 0 only the time history of the midpoint will be printed

G — Gravity constant (meters/sec^2)

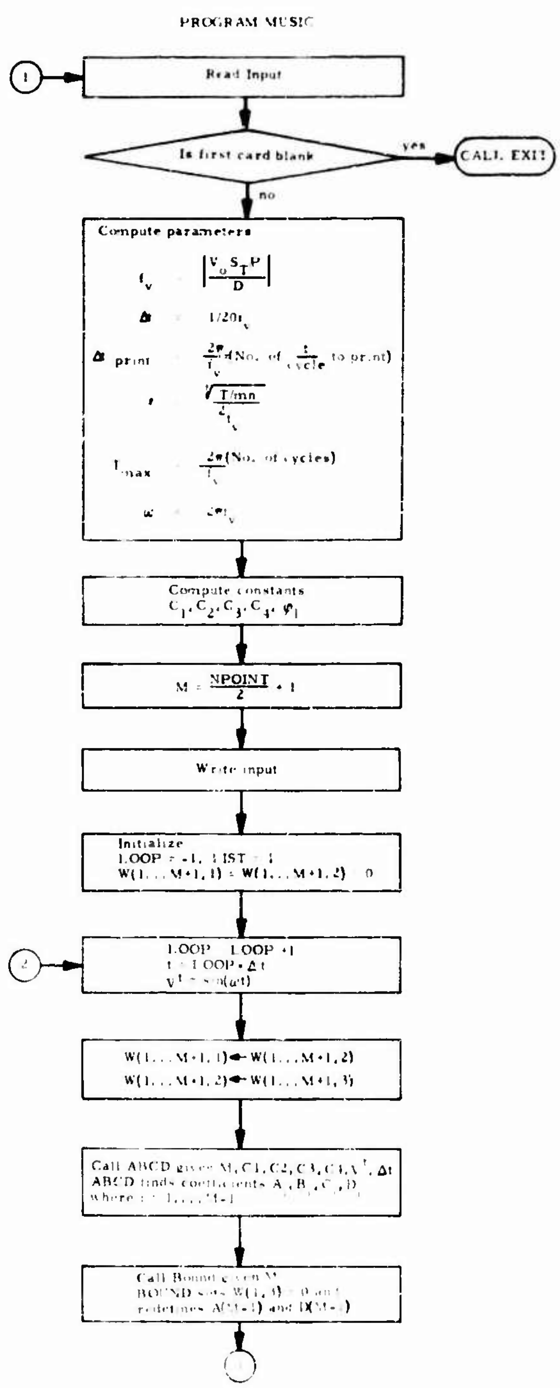
X — Altitude of the cable section (meters)

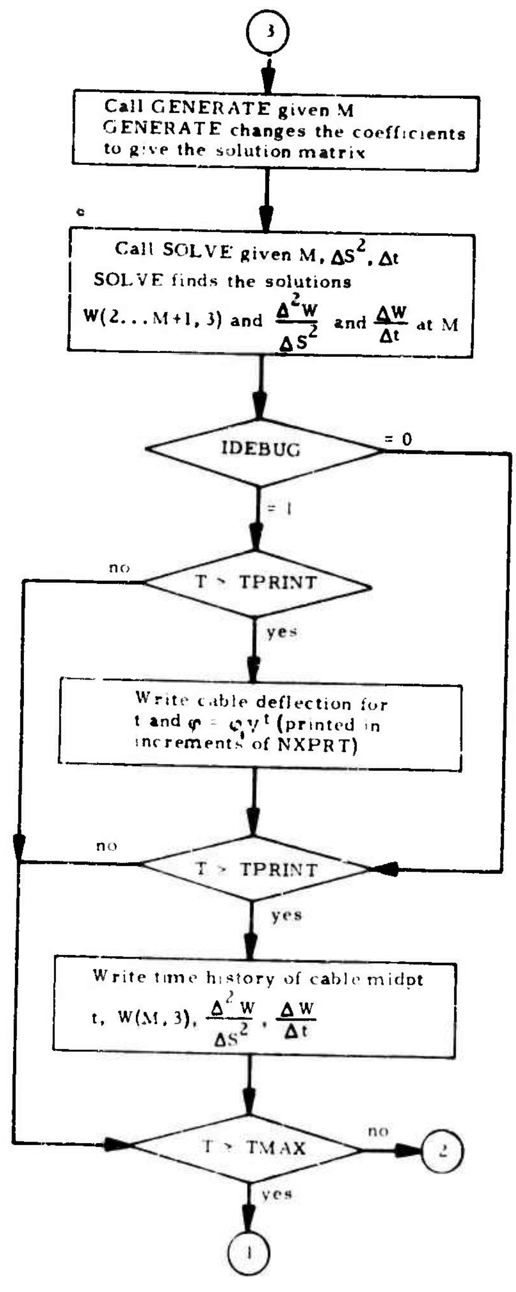
DS — ΔS = S increment between the points in the cable section (meters)

T — Tension in the cable section (kilograms-weight)

P	$P = \Delta x / \Delta S = \cos \alpha$ = determines slope of the cable section	
RHO	ρ = density of atmosphere (the program divides ρ by g) (Kg/meters^3)	
V0	V_o = wind velocity on cable (meters/sec)	
EM0	M_o = mass/unit length of cable (Kg/meter)	
D	Diameter of the cable (meters)	
C_D	Drag coefficient	
C_K	Lift coefficient	functions of Reynolds number
S_T	Strouhal number	

Flow Chart


PROGRAM MUSIC
Read Input
Is first card blank
yes
CALL EXIT
no
Compute parameters
Compute constants
$C_1, C_2, C_3, C_4, \varphi_1$
$M = \frac{NPOINT}{2} + 1$
Write input
Initialize
W(1...M+1, 1) = W(1...M+1, 2)
W(1...M+1, 1) ← W(1...M+1, 2)
W(1...M+1, 2) ← W(1...M+1, 3)
ABCD finds coefficients


3
Call GENERATE given M
GENERATE changes the coefficients to give the solution matrix
Call SOLVE given M, ΔS^2, Δt
SOLVE finds the solutions
W(2...M+1, 3) and $\frac{\Delta^2 W}{\Delta S^2}$ and $\frac{\Delta W}{\Delta t}$ at M
IDEBUG
= 0
= 1
no
T > TPRINT
yes
Write cable deflection for t and $\varphi = \varphi_i v^t$ (printed in increments of NXPRT)
no
T > TPRINT
yes
Write time history of cable midpt
t, W(M, 3), $\frac{\Delta^2 W}{\Delta S^2}$, $\frac{\Delta W}{\Delta t}$
T > TMAX
no
2
yes
1

```
      PROGRAM MUSIC
C     DYNAMIC ANALYSIS OF THE MUSICAL PROBLEM    SN344-716
      DIMENSION    W(100,3), A(100),B(100),C(100),D(100)
      COMMON W,A,B,C,D
C     NPOINT POINTS DESCRIBE A SHORT SEGMENT OF THE BALLOON CABLE
C     THE SEGMENT IS ANALYZED FOR M=NPOINT/2+1 POINTS USING A FICTITIOUS
C        POINT M+1
C     THE SHORT SEGMENT ASSUMES CONSTANT VALUES OF X,T,P,RHO,VO,CD,CK,ST
    1 READ (5,8000)NDA,NMO,NYR
 8000 FORMAT( 7I10)
      IF (NDA) 99,99,2
    2 READ (5,8000) NPOINT,NCYCLE,NXPRT,NTPRT,IDEBUG
      READ (5,8001) G,X,DS,       TEN,P,RHO,VO,EMO,DIA,CD,CK,ST
 8001 FORMAT (7F10.8)
C     TMAX OF THE RUN IS CALCULATED AS  NCYCLE.2PI/FREQUENCY FV
C     DT OF THE RUN IS CALCULATED AS 1/(20.FV)
C     THE CABLE SEGMENT LENGTH IS CALCULATED AS SQRT(T/MO)/(2.FV)
C     DATA IS PRINTED AT EVERY NXPRT POINT FOR EVERY DTPRT TIME
C     DTPRT=2PI/(FV.NTPRT)
      WRITE (6,9000) NDA,NMO,NYR
 9000 FORMAT (1H1,28X,50HDYNAMIC ANALYSIS OF THE MUSICAL PROBLEM--SN34M
     1716 ,4X,I2,1H/,I2,1H/,I2,//)
C     COMPUTE CONSTANTS
      CYCLES=NCYCLE
      FV=ABS(VO*ST*P/DIA)
      DT=0.05/FV
      TP=NTPRT
      DTPRT=6.2831853/(FV*TP)
      CABLE= 0.5*SQRT(TEN/EMO)/FV
      OMEGA=6.2831853*FV
      TMAX= 6.2831853*CYCLES/FV
      DT2=DT*DT
      DS2=DS*DS
      C1= 0.25*DT2*TEN/(EMO*DS2)
      C2= 0.25*RHO*CD*DIA*DT/(EMO*G)
      C3=(VO*P)**2
      PHI1= 0.5*RHO*CK*DIA*C3/G
      C4=PHI1*DT2/EMO
      M=NPOINT/2 +1
      MP=M +1
      WRITE (6,9001) NPOINT,M,CABLE,X,RHO,VO,EMO,DIA,TEN,P
 9001 FORMAT(19X,25H CABLE SEGMENT DEFINED BY,I3,24H POINTS AND ANALYZED
     1 FOR,I3,26H POINTS UP TO THE MIDPOINT,//,23H CABLE DATA     LENGTH
     2=,E15.8,5H   X=,E15.8,15H   DENSITY RHO=,E15.8,15H   VELOCITY VO=,
     3E15.8,/,15X,9H MASS MO=,E15.8, 13H  DIAMETER D=,E15.8, 12H  TENSIO
     4N T=,E15.8, 10H  P=DX/DS=,E15.8,/)
      WRITE (6,9002) CD,CK,ST
 9002 FORMAT(29H FUNCTIONS OF REYNOLDS NUMBER,15X,3HC =,E15.8,15X,3HC =,
     1E15.8,15X,3HS =,E15.8,/,44X,2H D,32X,1HK,32X,1HT,//)
      WRITE(6,9003) TMAX,DT,DTPRT,DS,G,FV
 9003 FORMAT(3X,13H MAXIMUM TIME,8X,13HTIME INTERVAL,4X,17HPRINTING INTE
     1RVAL,8X,7HDELTA S,13X,7HGRAVITY,11X,12HFREQUENCY FV,/,E17.8,5E20.8
     2)
C     INITIALIZE
      T=-DT
      TPRINT=DTPRT
      DO 100 I=1,MP
      W(I,2)=0.0
  100 W(I,3)=0.0
      WRITE (6,9015)
 9015 FORMAT (1H1,40X,38HTIME HISTORY OF CABLE SEGMENT MIDPOINT,//,27X,
```

```
     15H TIME,18X,1HW,13X,13H2ND DER DW/DS,11X,5HDW/DT)
      LOOP=-1
      LIST=1
C     RESET LOOP
  200 LOOP=LOOP+1
      SLOOP=LOOP
      T=SLOOP*DT
      VT=SIN(OMEGA*T)
      DO 300 I=1,MP
      W(I,1)=W(I,2)
  300 W(I,2)=W(I,3)
C     COMPUTE COEFFICIENTS A,B,C,D OF W SUCH THAT
C     A(M-1) W(M-1) + B(M-1) W(M) +C(M-1) W(M+1)=D(M-1)
      CALL ABCD(M,C1,C2,C3,C4,VT,DT)
C     ADJUST BOUNDARY CONDITIONS
      CALL BOUND (M)
C     GENERATE ELEMENTS IN THE SOLUTION MATRIX
      CALL GENERAT(M)
C     SOLVE THE SOLUTION MATRIX
      CALL SOLVE(M,D2WDS2,DWDT,DS2,DT)
      IF (IDEBUG) 450,450,350
  350 IF (T-TPRINT)450,400,400
  400 PHI=PHI1*VT
      WRITE (6,9010) T,PHI
 9010 FORMAT(  87X, 6H TIME=,E10.3,3X,4HPHI=,E10.3,/,97X,3H PT,13X,1HW)
      WRITE (6,9011)(I,W(I,3),I=1,M,NXPRT)
 9011 FORMAT (96X,I4,E20.8)
  450 IF (T-TPRINT) 500,460,460
  460 LIST=LIST+1
      SLIST=LIST
      TPRINT=SLIST*DTPRT
      WRITE (6,9016)    T,W(M,3),D2WDS2,DWDT
 9016 FORMAT(17X,4E20.8)
  500 IF (T-TMAX) 200,600,600
  600 WRITE (6,9020)
 9020 FORMAT(//,49X,22H *****END OF CASE*****)
      GO TO 1
   99 CALL EXIT
      END
      SUBROUTINE ABCD(M,C1,C2,C3,C4,VT,DT)
      DIMENSION    W(100,3), A(100),B(100),C(100),D(100)
      COMMON W,A,B,C,D
      DO 100 I=2,M
      MM=I-1
      MP=I+1
      A(MM)=-C1
      CON=C2*SQRT(C3+(((W(I,2)-W(I,1))/DT)**2) )
      B(MM)=1.0-2.0*A(MM)+CON
      C(MM)=A(MM)
      D(MM)=-A(MM)*(W(MP,1)+W(MM,1)-2.0*W(I,1)+2.0*(W(MP,2)+W(MM,2)
     1-2.0*W(I,2)))+C4*VT+CON*W(I,1) + 2.0* (W(I,2)-W(I,1))
  100 CONTINUE
      RETURN
      END
      SUBROUTINE BOUND(M)
      DIMENSION    W(100,3), A(100),B(100),C(100),D(100)
      COMMON W,A,B,C,D
      W(1,3)=0.0
      MM=M-1
      MP=M+1
      A(MM)=A(MM)+C(MM)
```

```
      D(MM)=D(MM)+C(MM)*(W(MP,1)-W(MM,1)+2.0*(W(MP,2)-W(MM,2)) )
      RETURN
      END
      SUBROUTINE GENERAT(M)
      DIMENSION    W(100,3), A(100),B(100),C(100),D(100)
      COMMON W,A,B,C,D
      C(1)=C(1)/B(1)
      D(1)=D(1)/B(1)
      MM2=M-2
      DO 100 I=1,MM2
      MP=I+1
      B(MP)=B(MP)-A(MP)*C(I)
      C(MP)=C(MP)/B(MP)
      D(MP)=(D(MP)-A(MP)*D(I))/B(MP)
  100 CONTINUE
      RETURN
      END
      SUBROUTINE SOLVE(M,D2WDS2,DWDT,DS2,DT)
      DIMENSION    W(100,3), A(100),B(100),C(100),D(100)
      COMMON W,A,B,C,D
      MM=M-1
      MP=M+1
      W(M,3)=D(MM)
      DO 100 I=2,MM
      J=MM-I+2
      W(J,3)=D(J-1)-C(J-1)*W(J+1,3)
  100 CONTINUE
      DWDT     = 0.5*(W(M,3)-W(M,1))/DT
C     SYMMETRICAL CONDITIONS AT POINTS M-1 AND M+1
      W(MP,3)=W(MM,3)-2.0*(W(MP,2)-W(MM,2))-W(MP,1)+W(MM,1)
      D2WDS2=0.25*(W(MP,3)+W(MM,3)+W(MP,1)+W(MM,1)+2.0*(W(MP,2)+W(MM,2)
     1 -W(M,3)-W(M,1)-2.0* W(M,2)))/DS2
      RETURN
      END
```

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A computer study has been made of nonsteady aerodynamic loadings on a long, cylindrical cable of the continuous glass fiber-resin type used as a tether for a balloon at altitudes of approximately 100,000 feet. No important interactions between torsional, longitudinal, and lateral modes were found. Furthermore, the effects of lateral loadings from gusts or wortex sheddings were found to be unimportant. Computer programs are presented that enable computations to be made of cable motions resulting from localized gust loadings and from vortex shedding phenomena.

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